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An experiment to determine gravity from geoid heights in Turkey.

by

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Abstract: Point and mean gravity values have been computed for Turkey using geoid heights digitized from a map. Least squares collocation (LSC) and the Fourier method (FFT) was used to convert the geoid heights from which the contribution from OSU91A had been subtracted to mean and point gravity values. A covariance analysis showed that the residual (i.e. free-air minus OSU91) gravity anomalies had a standard deviation of ±33 mgal. Corresponding 6' x 10' mean gravity values had a standard deviation of ±27 mgal. The minimum LSC estimated error of the mean values were ±11 mgal, based on a standard deviation of the error in the digitized heights equal to ±0.2 m. LSC and FFT derived point values agreed within ±7 mgal (standard deviation).

The values were compared with 94 point values in South-West Turkey, where gravity values and topographic data had been used to compute the geoid heights. The observed and predicted values had differences with a standard deviation equal to ±37 mgal, which is larger than the expected signal standard deviation. This is probably due to the strong geoid variation in the area. At sea, where no bathymetric data had been used to construct the geoid, the difference had for 362 values a mean value equal to -40 mgal and a standard deviation of ±49 mgal.

The obtained results are not satisfactory. However, improved results could be obtained if the digital terrain model used when calculating the geoid also could be used in a remove-restore procedure for gravity recovery.

1. Introduction.

The geoid may be determined from gravity values, and if the geoid is known, we may obtain the gravity values from the geoid. For the GEOMED-project, which aims at calculating a precise geoid for the Mediterranean area, a uniform gravity coverage is needed as basic data. In principle, geoid values could be mixed with gravity values, but the computational procedures necessary to do this (least squares collocation, LSC) is very demanding on computer resources. Also, the low precision of the global gravity models (especially) in the (Eastern) Mediterranean requires that the data in large areas is treated simultaneously in the computations.

A precise geoid has recently (Ayhan, 1991) been computed for Turkey. It is based on 62250 gravity values, which unfortunately could not be released even for scientific purposes. The remove-restore method for topographic effects (Forsberg & Tscherning, 1981) was used. And the global geopotential model (GPM2, Wenzel,(1985)) was updated using the known gravity values. The method of stepwise LSC was used for the calculation. A comparison with geoid height differences obtained from GPS and levelling showed that the geoid had a very good quality.

With this available, it should be possible to compute mean gravity from the geoid heights. Point gravity values should, as we also will se later, not be very well determined considering that we also did not have available the digital terrain model used by (Ayhan, 1993). We therefore decided to follow the normal computational procedures used when applying LSC. They will be described in the following section 2.

A few gravity values are available for the Turkish area and for the adjacent Mediterranean. They have been used to check the result, as we will see in section 3 with a rather negative result.

2. LSC computation of gravity anomalies for Turkey.

First the geoid map, which is bounded by 35.6° and 42.2° in latitude and 26° and 44.8° in longitude was digitized in a 0.2° x 0.2° grid, i.e. totally 3230 values. We judge that the digitized values have an error of ±0.2 m. From these values we subtracted the contribution from the OSU91A spherical harmonic expansion (Rapp et al., 1991), which is complete to degree and order 360.

The differences were used to compute an empirical geoid height covariance function. This function was used to derive an analytic covariance model using the program COVFIT (see Knudsen, 1987). The empirical and the analytic covariance function is shown in Figure 1.

The analytic expression for the gravity anomaly covariance function for two gravity anomalies in points P, Q, had the following form

$$cov_{\Delta g}(\psi,r,r') = \sum_{i=2}^{28} a\sigma_i^e \left(\frac{R^2}{rr'}\right)^{i+2} P_i(\cos\psi) + \sum_{i=29}^{\infty} \frac{A\ (i-1)}{(i-2)(i+4)} \left(\frac{R^2}{rr'}\right)^{i+2} P_i(\cos\psi)$$

where ψ is the spherical distance, r and r' are the distances from the origin for P, Q, respectively, R is the radius of the Bjerhammar-sphere and σ_i^e are the error-degree-variances of the OSU91A spherical harmonic coefficients a a scale factor equal to 0.09, $A{*}R_E^2$ = 204.42 mgal²i and R_E is the mean radius of the Earth. The depth to the Bjerhammar sphere (R_E-R) was 250 m. The point gravity standard deviation at zero altitude becomes with this model equal to 33.2 mgal. Note that the summation index for the error degree-variances terminates at degree 28. This is an indication of that only the information in the spherical harmonic coefficients up to this degree is valid for the Turkish area !

Totally 2698 observations in the area bounded by 36° and 42° in latitude and 26.8° and 44° in longitude were used in the LSC computation using the program GEOCOL (Tscherning, 1992). The program was also used to calculate 6' x 10' mean gravity anomalies and their error estimates, as well as point gravity anomalies in points where

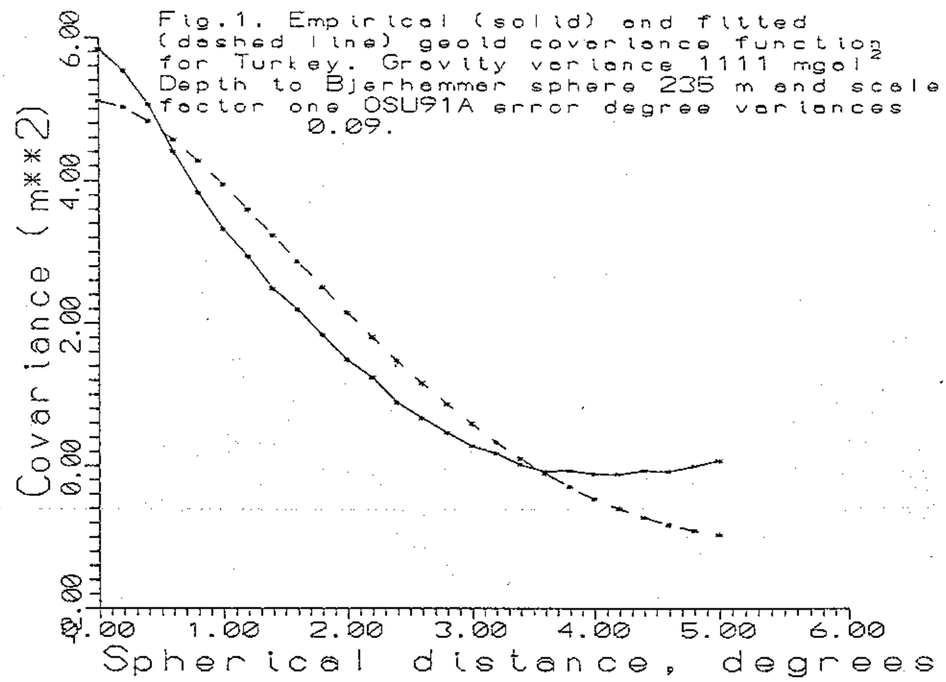

Fig.1. Empirical (solid) and fitted (dashed line) geoid covariance function for Turkey. Gravity variance 1111 mgal^2 Depth to Bjerhammer sphere 235 m and scale factor one OSU91A error degree variances 0.09.

the gravity was known. The computations which were executed on an Apollo DN10000 lasted more than 24 hours.

The error estimate was in the best case (the middle of the area) equal to 11 mgal. If this value is correct (which we shall discuss in a moment) then the result would have been useful for geoid computation. The computed (residual) gravity values are shown in Fig. 2. The mean free-air anomaly is then obtained by adding the contribution from the OSU91A spherical harmonic expansion, see Figure 3.

The data were also used to compute gravity values using the Fourier method, see Schwarz et al. (1990). The program GEOFOUR was used for this purpose. The computations lasted in this case less than a minute.

The result was compared with the result of LSC and a good agreement was obtained. The differences between the two results had a mean value of 3.7 mgal and standard deviation of ±7.5 mgal. However, the maximal and minimal differences were -35 and 77 mgal, respectively.

In order to check the solution, point gravity values were computed in points with known values. From D.Arabelos, Thessaloniki, 2 datasets were obtained. One with 96 land gravity values from South-West Turkey and the other with Sea-gravity from the adjacent Mediterranean. The result is shown in Table 1.

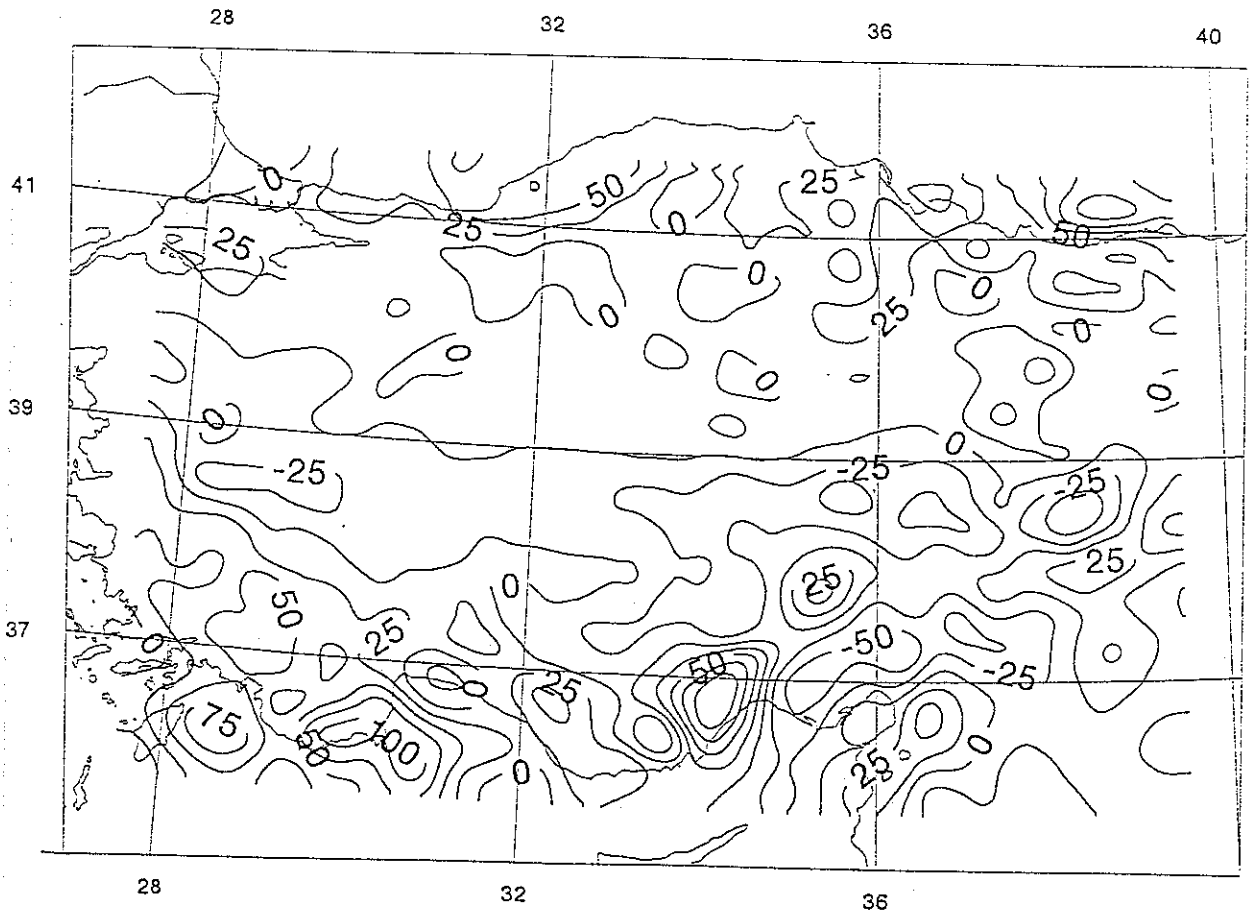

Figure 2. 6'x10' mean residual gravity from geoid heights minus OSU91A.

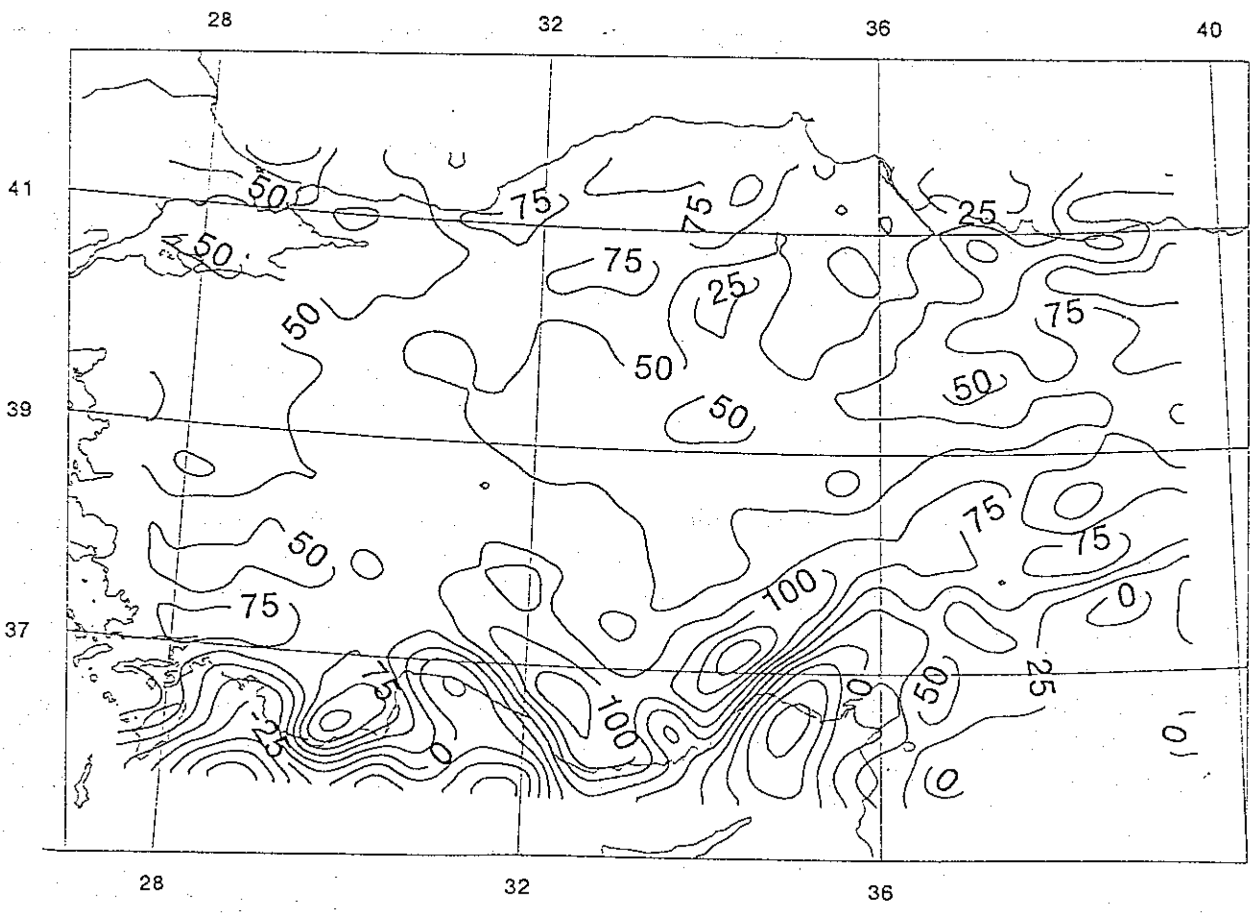

Figure 3. 6' x 10' mean gravity in Turkey from geoid heights.

Table 1. Prediction results. All units mgal.

	Original data			Predicted		Difference Pred.-Original	
	Number	Mean	Stdv.	Mean	Stdv.	Mean	Stdv.
Land	96	17.4	42.5	38.0	37.6	-20.7	37.7
Sea	362	-55.4	65.1	-15.7	49.8	-39.7	49.3

The results are not good. The differences have standard deviations of the order of the signal standard deviation. The difference in mean value is probably due to an erroneous reference system used (Potsdam).

A partial explanation for the large differences are the following. For the land data we have used data in an area with a strongly sloping geoid. The digitization procedure in this area was probably not good enough. Also, the map used for digitization lacked meridians and parallels!

For the sea-data we have here a result, where the geoid was computed without the use of bathymetry. This may explain why we do not recover the high frequency information.

3. Conclusion.

The computed gravity values have formal standard deviations (error estimates) which indicates that the result is usable for geoid computation. However, the comparison with true values show that this may not be correct. An improved result requires the use of a digital terrain model, and of the gravity data which is freely available.

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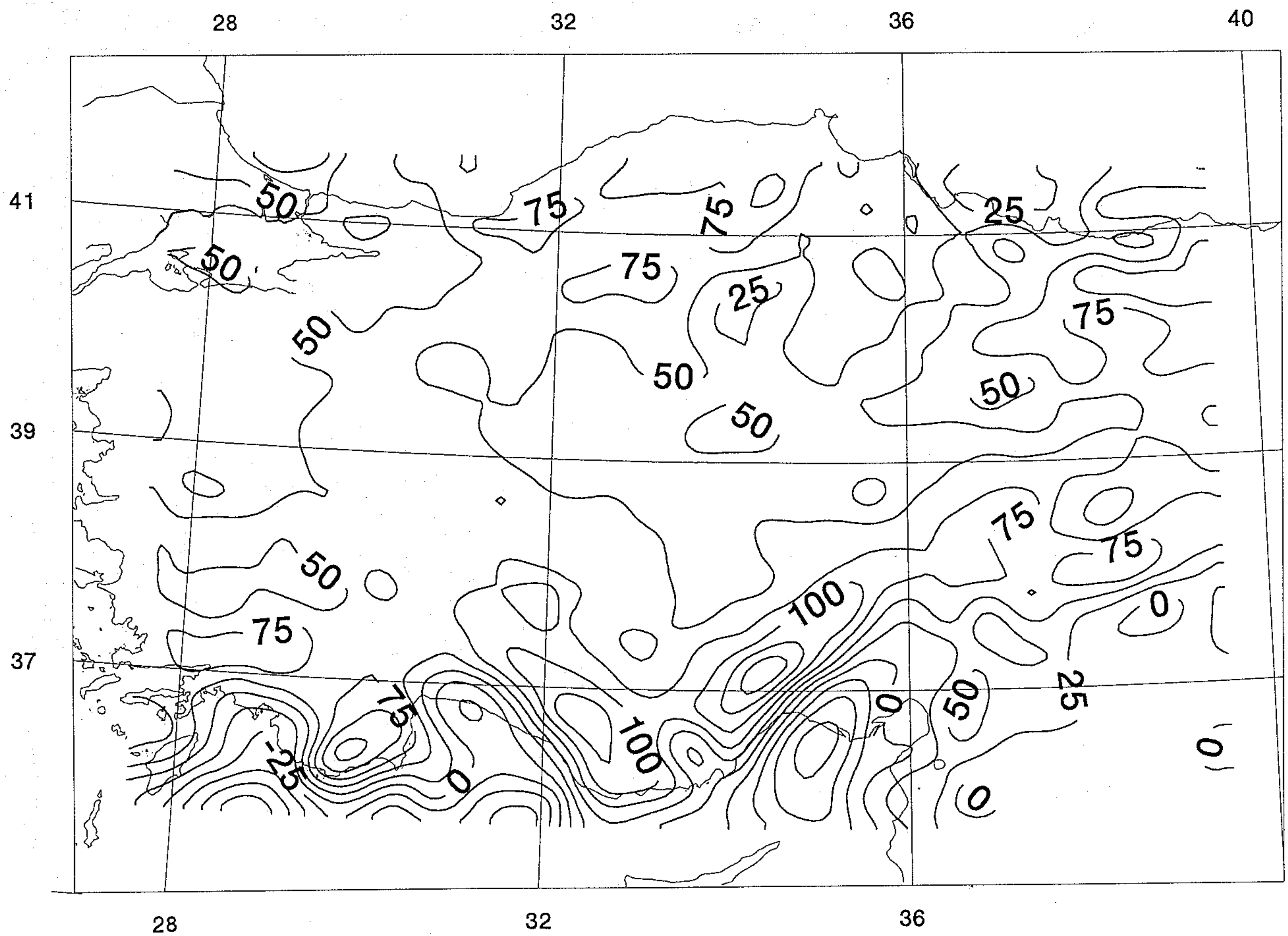

Figure 3. 6' x 10' mean gravity in Turkey from geoid heights.

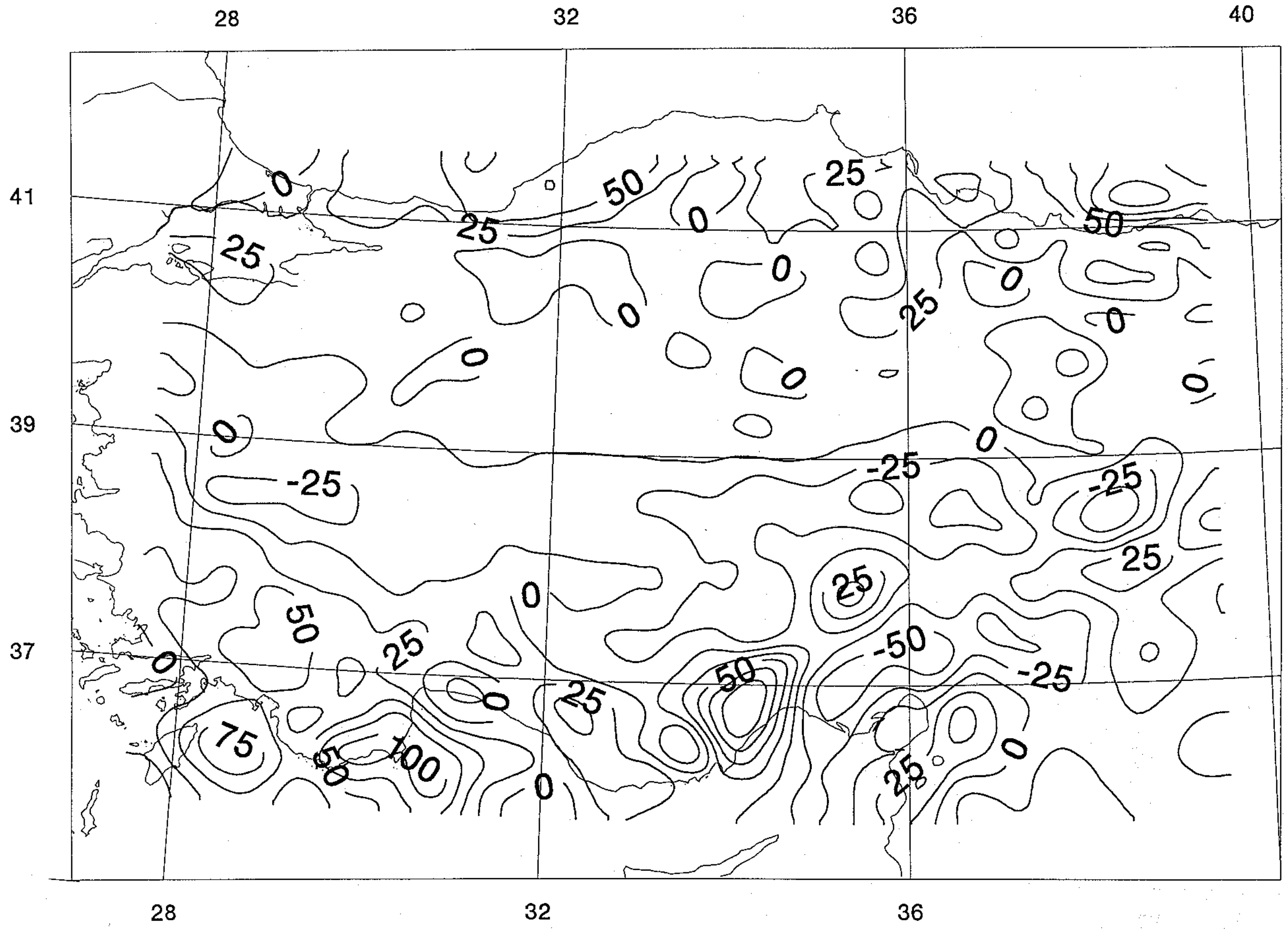

Figure 2. 6'x10' mean residual gravity from geoid heights minus OSU91A.