GEOID COMPUTATION IN THE MEDITERRANEAN AREA

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1. Introduction

This paper deals with the computation of a gravimetric geoid on a large a covering the Western part of the Mediterranean Sea. This is done in framework of the Geomed project which aims at evaluating a high precis geoid in the whole Mediterranean area. What we have done can be considered a first experiment suitable to define the numerical and theoretical methods be applied on the global area. The techniques we used to estimate the ge undulation from gravity data are Stokes/FFT formula (Sideris 1987, Forsb and Solheim, 1988) and Fast Collocation (Bottoni and Barzaghi, 1991). Th allowed us to obtain the solution in one step only, overcaming the patching local solutions computed on a partition of the area under investigation. first important comparison we derived is the one on the two different meth we applied on the same gravity data base; to the knowledge of the autho this is the first time that an FFT derived geoid is compared with collocation solution computed in one step on such a large area. A second che on the estimated geoid has been obtained by comparison with the adjust altimetry data (both from GEOSAT and SEASAT satellites). A byproduct of th is an estimate of the Sea Surface Topography (SST) which gave us a fir insight on the water circulation pattern in the western part of t Mediterranean Sea.

2. Some remarks on geoid estimation

The method we used to estimate the geoid is the remove-restore technique (Forsberg, 1985, Tscherning, 1985, Arabelos and Tscherning, 1988). As it is well known, while using such a procedure one splits the geoid computation in the following steps:

remove the effect of a global geopotential model from the data;

remove the Residual Terrain Effect (RTE) from the data, i.e. the effect implied by the masses between the topography and a proper, smooth reference topography (usually obtained from the detailed digital terrain model applying a proper low-pass filter);

estimate, using the reduced data, the residual geoid using one method or another (collocation, Stokes and so on);

restore the RTE in terms of geoid undulation, adding this effect to the residual geoid;

restore the geoid undulation implied by the global geopotential model.

This method, as it is evident, performs a frequency analysis of the data, removing the low frequency part (the geopotential model) and the high frequency component (RTE). Removing the global geopotential model localizes the data so that only the correlations between points in the local area are significant; the subtraction of the RTE regularizes strongly the data decoupling these from the main topographic features and their high frequency signal. So, the reduced signal should have suitable statistical propreties (nearly zero mean and small variance) and should not contain any relevant trend; consequently, collocation (or else) gives an optimal estimate of the residual geoid. Although this procedure is nowadays commonly used, there are still some critical points in its practical application. As pointed out before, the global geopotential models should remove the low frequency components from the data; unfortunately, in some regions, these geopotential models contain biases and do not represent properly the gravity field. This is the case of the southern and the eastern Europe areas because of the lack of data in the eastern Europe nations and in Africa.

Furthermore, the methods used to compute the coefficients of the spherical harmonic series of the Earth gravitational potential lead to biased estimates (Albertella and others, 1981). Thus, the subtraction of the globl geopotentaial model can introduce significant distorsions, that is non zero

mean residuals and/or no suitable modelling of the trend present in the r
data.

The second step of the remove-restore method can give rise to problems too.
There are no precise rules to define the smooth reference terrain model
compute the RTE. Since it is supposed that the global geopotential mo
accounts also for the low-frequency part of the topographic signal, one
confronted with the problem of defining the proper smoothed version of
topography, which should be the one implied by the geopotential model.

However, as noticed before, the geopotential models are in some ca
unreliable and sometimes poorly correlated with the main topographic featur
If we consider them from a local point of view, phase shifts can occur betw
observed and model implied values, causing problems both in trend removal
RTE computation.

The definition of the reference surface for RTE computation is crucial; o
if we define it properly we can reduce the data in a coherent w
producing the expected regularization of the data. In pratical applicati
different rules of thumb are adopted but no theoretical procedure has b
devised. Hence, we cannot be sure that the second step of the remove proced
is correctly done because we cannot state that the reference smooth topogra
matches with the selected geopotential model (see fig. 1.)[1].

This can give problems when areas of about 2^0 of size are considered; if
topography in the area is rough and the surface implied by the model diff
from the one used for the RTE computation, a significant medium freque
trend can remain in the residual values. This trend is obviously related
the effect of the masses between the two reference surfaces mentioned bef
(see fig 1.) and can compromise the next step of the geoid estimation (if, i
instance, we use collocation on the residuals and a trend is still present,
can have distorsions in the geoid estimate). In the numerical tests we did
the Mediterranean Sea, we selected a quite large area ($35^0 \leq \phi \leq$ 4
; $-5^0 \leq \lambda \leq 10^0$); thus, the problems due to the removal of a geopotential
model effect and to an improper RTE reduction, in the sense specified befor
should not introduce a so relevant trend that collocation cannot handle. Henc
we simply averaged the higher resolution DTM to obtain the smooth referen

[1] A possible correct procedure could be to vary the reference surface unti
the residual signal, i.e. data minus the model and the RTE contributio
decorrelates with the geopotential model

surface used for the RTE computation.
The results we got, seem to confirm that, in our case, the problems related to the model and to the RTE computation are not critical.
The final remark refers to the gridding of the data. Since we applied Stokes/FFT and the Fast Collocation methods, we needed gridded homogeneous data. We applied the gridding procedure to the smoothed gravity data, that is to

$$\Delta g_r = \Delta g_0 - \Delta g_M - \Delta g_{RTE} \qquad (2.1)$$

to minimize gridding errors. It is in fact evident that Δg_r data can be interpolated in a more accurate way than Δg_0 bacause of their optimal spectral properties (i.e. only medium wave components).

3. The gravimetric geoid computation

In the Western Mediterranean area we have a good data coverage. In the selected window

$$35^0 \leq \phi \leq 45^0$$
$$-5^0 \leq \lambda \leq 10^0 \qquad (3.1)$$

there are 7944 marine gravity data (placed on a regular grid of 5'x5') and 28837 gravity data on land.
The marine gravity data were supplied by D. Arabelos (1988) while the terrestrial gravity data comes from the BGI or from the data base of the research group involved in the GEMED project; terrestrial gravity data are

subdivided according to tab. 1.

Nation	N. of data
ALGERIA	1738
FRANCE	5196
ITALY	1010
MAROCCO	433
SPAIN	20209
TUNISIA	51

Tab. 1. - Avaliable gravity data on land

The statistical properties of these data are summarized by the figures

$$E(\Delta g) = 11.12 \text{ mgal}$$

$$\sigma(\Delta g) = 31.51 \text{ mgal}$$

and the contour line plot of these data is represented in fig. 2.
As one can see, there are sharp variations in the terrestrial grav
mainly related to the topography of the area which is quite rougl
north-west corner (Pyrenees Mountain chain) and close to the southe
(Altante Mountain chain). On the contrary, the sea floor is quite smc
fig. 4) and this reflects in the feature of the marine gravity data
this heterogeneity, the remove-restore technique becomes an irrep
method to get a good geoid estimate. Furthermore, the problems relate
remove steps pointed out previously should not be relevant as
selected area is quite large; hence the computation can be carried out
any further refinement of the remove-restore procedure.

We firstly subtracted the gravity effect of a global geopotential model; t
one we used for the computation is the OSU91A by R.H. Rapp (1991) plotted
fig. 3.

To model and remove the RTE we used the TUG87 terrain model (Weiser, 1987
which gives the topography/bathymetry of the Earth on a regular grid of 5'x5
The smooth reference topography was computed from the TUG87 with a movi
average of 15'x15'; TUG87 and its smoothed version in the selected area a
plotted in fig.4 and fig.5.

To compute the RTE we used the TC program by R. Forsberg which gives t
gravity terrain component using the rigorous formula of the gravity effect
a parallelepiped; this correction was obtained considering all t
parallelepipeda with centers contained in a circle centered on the computati
point with radius 100 km.

At the end of the remove procedure, we had the residual gravity of fig. 6.
As expected, no relevant trend are present, the main topograph
irregularities disappeared, the dominant frequency pattern of the residu
gravity is a medium one and the signal has been smoothed significantly. Th
can be also deduced from table 2 where the statistics of the remo
procedure are listed (note that the number of data decreases because,
the processing procedure, outliers have been rejected).

	N. of data	Average (mgal)	St. dev. (mgal)
Δg	36581	11.12	31.51
$\Delta g - \Delta g_{OSU91A}$	36570	-3.92	25.88
$\Delta g_r = \Delta g - \Delta g_{OSU91A} - \Delta g_{RTE}$	36570	0.35	16.97

Tab. 2. - Remove statistics

The gridding procedure has been applied on Δg_r using the GE
R. Forsberg, performing a spline interpolation and produ
geographical grid with grid side of 5'. Thus, in the selected
17557 gridded gravity data Δg_r^G with the following statistical

$$E(\Delta g_r^G) = 1.22 \text{ mgal}$$

$$\sigma(\Delta g_r^G) = 14.50 \text{ mgal}$$

Note that the gridding procedure leads to a gravity data
statistically close to the original Δg_r data set; this suggests
of the choice of gridding the smooth Δg_r values.
To start with the collocation procedure to get the residual geoid
the empirical covariance function of Δg_r which is plotted in fig.
As one can notice, the correlation length is nearly 15' confirm
doesn't contain long wavelength tends. The selected best fit mod
obtained using thr COVFIT program by Knudsen (1987), was of the fo

$$C(P,Q) = C(\psi_{PQ}) = \sum_{i=1}^{m} \sigma_i^e \left[\frac{R^2}{r_P r_Q} \right]^{n+2} P_n(\cos \psi_{PQ})$$

$$+ \sum_{i=m+1}^{\infty} \sigma_i \left[\frac{R^2}{r_P r_Q} \right]^{n+2} P_n(\cos \psi_{PQ})$$

where σ_i^e are the error-degree variances used up to m=120 and

$$\sigma_i = \frac{(i-1)^2}{R^2} A \frac{(1-1)}{(i-1)(i-2)(i+24)}$$

R = mean Earth radius

The selected depth of the Bjerhammar sphere was $R_B - R = -2393.725$ m an
the noise variance level was fixed to 4 $mgal^2$.
The output of Stokes/FFT and Fast Collocation are practically the same (s
fig. 8 and fig. 9). This good fitting between the two estimates is confir
by the correlation coefficient $\rho = 0.98$; hence, the comparison between the
procedure leads to a very encouraging result. The main difference in
solutions is a bias; the collocation solution is 45 cm higher than
Stokes/FFT derived undulation (which has zero average - see tab. 2).
discrepancy in the mean can be explained taking into account that in
Stokes/FFT procedure we removed the average of the gravity data ($E(\Delta g$
1.22 mgal) and that we didn't do the same for the collocation procedure.
was done, in the Stokes/FFT method, to damp the high frequency part of
spectrum of the data induced by the non zero average which can c
instabilities in the computing procedure. On the contrary, as this is
strictly necessary, we didn't remove the average in the collocation proce
trying, in this way, to overcome the "restore of the mean problem". Thi
another critical point in the whole estimation procedure; if we remove
mean from the gravity data, we cannot restore it in term of geoid undul
since, as it is well known, the potential implied by a constant gr
value tends to infinity in the planar approximation. So, rigorously spea
we can only compute relative geoid undulation since, by differentiatir
remove a common undefined bias. Furthermore, we stress that this p
becomes crucial when estimating a regional geoid by patching local solu
If the local gravity data have different mean values, the local estimate
differ by uneven local constants which will not be restored properly
exactly through this mechanism that patching biased local solution
result in a medium wavelength distorsion of the global geoid, as revea
comparisons with GPS/levelling geoid.
In our case, since we computed directly the solution on the whole ar
overcame the patching problem and, taking into account in the futher s
the restore procedure the collocation estimate of N_r, we disregarde
"restore of the mean problem" discussed before; we simply let coll
compensate for the small non zero mean of Δg_r^G. This in fact is li
happen when, in front of a grid spacing of 5', we have a process
correlation length as it in this case. Since the method is stable
residual gravity data average is nearly zero, we can hopefully assume

relevant distorsions are present in the estimated residual geoid; hov
be cautious, we can say that we estimated, also in this case, a
geoid.

The restore statistics are summarized in table 3 while the plots of
RTE geoid, and of N_M(OSU91A) , the geopotential model implied geoid,
$\hat{N}_r$(FC), are represented in fig. 10 and fig. 11; the astimated geoid is
in fig. 12.

	N. of data	Average (m)	St. d (m)
$\hat{N}_r$(FFT)	17557	0.00	0.66
$\hat{N}_r$(FC)	17557	0.45	0.67
$\hat{N}_r$(FC) + N_{RTM}	17557	0.54	0.67
$\hat{N} = \hat{N}_r$(FC) + N_{RTM} + N_{OSU91A}	17557	47.99	3.40

Tab. 3. - Restore statistics

4. The comparison between the gravimetric geoid and the adjusted alt data; Sea Surface Topography estimation.

The altimetric observations give the height of the satellite over th
surface, measuring the travel time of a radar pulse. Assuming to
the satellite coordinates in a geocentric reference frame, one can compu
height H_{alt} of the sea surface over the mean earth ellipsoid. By defi
this quantity can be splitted up in two components, i.e. the geoid N a
SST, ζ

$$H_{alt} = N + \zeta \qquad (4.$$

where ζ is the sum of two terms, the stationary ζ^0 and the time varying SST. Hence, in principle, one can estimate ζ using H_{alt} and an estimate of N This very simplified scheme to compute ζ doesn't take into account the mo errors of the altimetric measurement.

The main problem is related to the position of the satellite which is kno with a certain degree of error due, to approximations in the estimation of satellite orbit. As it is evident, the radial orbit error is critical w dealing with altimetric observation. This error however becomes visible cross-over discrepancies, i.e. differences of heights between crossing tra (southgoing and northgoing tracks); to compensate for radial obit error, cross-over adjustment procedure is adopted (Rowland, 1981).

For a local area, as the one we are dealing with, the radial orbit error modelled using a straight line; the cross-over adjustment allows to estimat track by track bias and tilt, imposing the cross-over consistency conditi The rank deficiency related to this least squares problem (Schrama 19 Barzaghi and others, 1992) can be removed, for instance, combining cross-o equations and fit equations of the along track observations to a model ge (e.g. N_{OSU91A}). At the end of the adjustment procedure, we have the $\hat{H}$ values which are suited to estimate $\hat{\zeta}$ by comparison with $\hat{N}$. One point t must be stressed is that the radial orbit error estimate and removal produ a distorsion in the estimate of ζ. The components of ζ having the same sh of the radial orbit error are modelled at the same time in the bias and t analysis. Hence, this component will be removed in $\hat{H}_{alt}$ and in the derive quantity.

Following the procedure previously described, we adjusted the altimet observations we had in the area, consisting of 3971 SEASAT values (arranged 56 tracks, giving 254 cross-overs) and of 11062 GEOSAT values (arranged in 2 tracks, giving 1956 cross-overs).

The cross-over condition was combined to the fit equation of each track w the OSU91A model geoid; separate adjustments were firstly performed on SEA and GEOSAT data. The statistics of the cross-over discrepancies

$$v_{ij} = H^i_{alt} - H^j_{alt}$$

and of the discrepancies with the model

$$\Delta_i = H^i_{alt} - N^i_{OSU91A}$$

both for SEASAT and GEOSAT, before and after the adjustment procedu
compared in table 4.

		v^o	Δ^o	$\hat{v}$	$\hat{\Delta}$
SEASAT	E (m)	-0.01	1.24	0.00	0.00
	σ (m)	0.20	0.52	0.06	0.39
GEOSAT	E (m)	-0.82	-0.07	0.00	0.00
	σ (m)	0.30	0.62	0.04	0.31

Δ^o, v^o = unadjusted values $\hat{\Delta}$, $\hat{v}$ = adjusted values

Tab. 4. - Cross-over adjustment results

Th statistics of table 4 prove that rough GEOSAT data fit N_{OSU91A} b
than rough SEASAT data. The converse is true for the cross-over consiste
rough GEOSAT observations which is poorer than the one derived from
observations. This features are simple consequences of the dif

preprocessing applied to the two altimetric data sets.
The SEASAT values we have were locally adjusted, for biases only, on t
Mediterranean Sea (Cruz and Rapp, 1982) while GEOSAT data were global
adjusted (Denker and Rapp, 1990).
Since, the processed values are quite omogeneous, we decided to merge t
altimetric observations from the two satellite in a global adjustment (148
values with 3704 cross-overs). The statistics related to the global set
data are the following

$$E(\hat{v}) = 0.00 \text{ m} \qquad \sigma(\hat{v}) = 0.05 \text{ m}$$

$$E(\hat{\Delta}) = 0.02 \text{ m} \qquad \sigma(\hat{\Delta}) = 0.36 \text{ m} \qquad (4.$$

and the corresponding $\hat{\Delta}$ are plotted in fig. 13.
To obtain the estimate of $\hat{\zeta}$, we interpolated the gridded $\Delta\hat{N} = \hat{N}_{collo} + \hat{N}_{RTM}$
values along the satellite tracks (note that the same model N_{OSU91A} was omitt
here as in the $\hat{\Delta}$ values from altimetry) obtaining

$$E(\Delta\hat{N}) = 0.61 \text{ m} \qquad \sigma(\Delta\hat{N}) = 0.42 \text{ m} \qquad (4.$$

The estimate of the SST

$$\hat{\zeta} = \hat{\Delta} - \Delta\hat{N} \qquad (4.$$

is shown in fig. 14; the mean and the standard deviation of the $\hat{\zeta}$ are

$$E(\hat{\zeta}) = -0.59 \text{ m} \qquad \sigma(\hat{\zeta}) = 0.32 \text{ m} \qquad (4.$$

The main features of the SST are a quite picked minimum south-east o
Balearic Islands and a global trend placed nearly along the $\phi = 38^0$ par

5. Conclusions

The comparison between Stokes/FFT and Fast Collocation solutions gave
result so that one of the aims of the present paper has been reached.
Furthermore, the whole remove-restore technique has been critically rev
view of its application to the geoid estimation in the whole Medite
area. Although some problems are still unsolved from the theo
standpoint, this procedure led to a proper geoid estimate as one can i
comparison with altimetry.
The geoid we computed is by no means a high precision geoid. A more
validation of data must be carried out and a more detailed DTM should
in the RTE computation. In fact, it is evident that the 5' x 5' TUG87
model is too poor to account for the high frequency pattern of th
Another crucial point is the data acquisition; large areas
Mediterranean region have no sufficient data coverage. So, to attain th
goal of the GEOMED project, we shall have to fill in data gaps and ac
more detailed DTM.
Nevertheless, this first geoid estimate proved the feasibility of this
and, as a first by-product, an estimate of the SST has been derived.
Future more accurate estimates of the geoid and of the SST will a
compared to altimetry data of the ERS1 satellite.

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Index of illustrations

Fig. 9. - Fast Collocation residual geoid (m)

Fig. 10. - RTE geiod (m)

Fig. 11. - Geoid undulation implied by the geopotential model OSU91A (m)

Fig. 12. - The estimated gravimetric geoid (m)

Fig. 13. - Adjusted altimetry (GEOSAT and SEASAT data) minus OSU91A model geoid (m)

Fig. 14. - Sea Surface Topography (m)

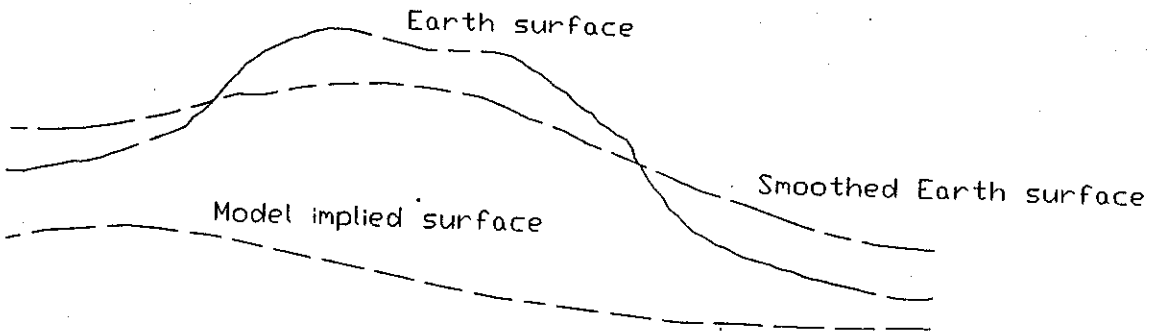
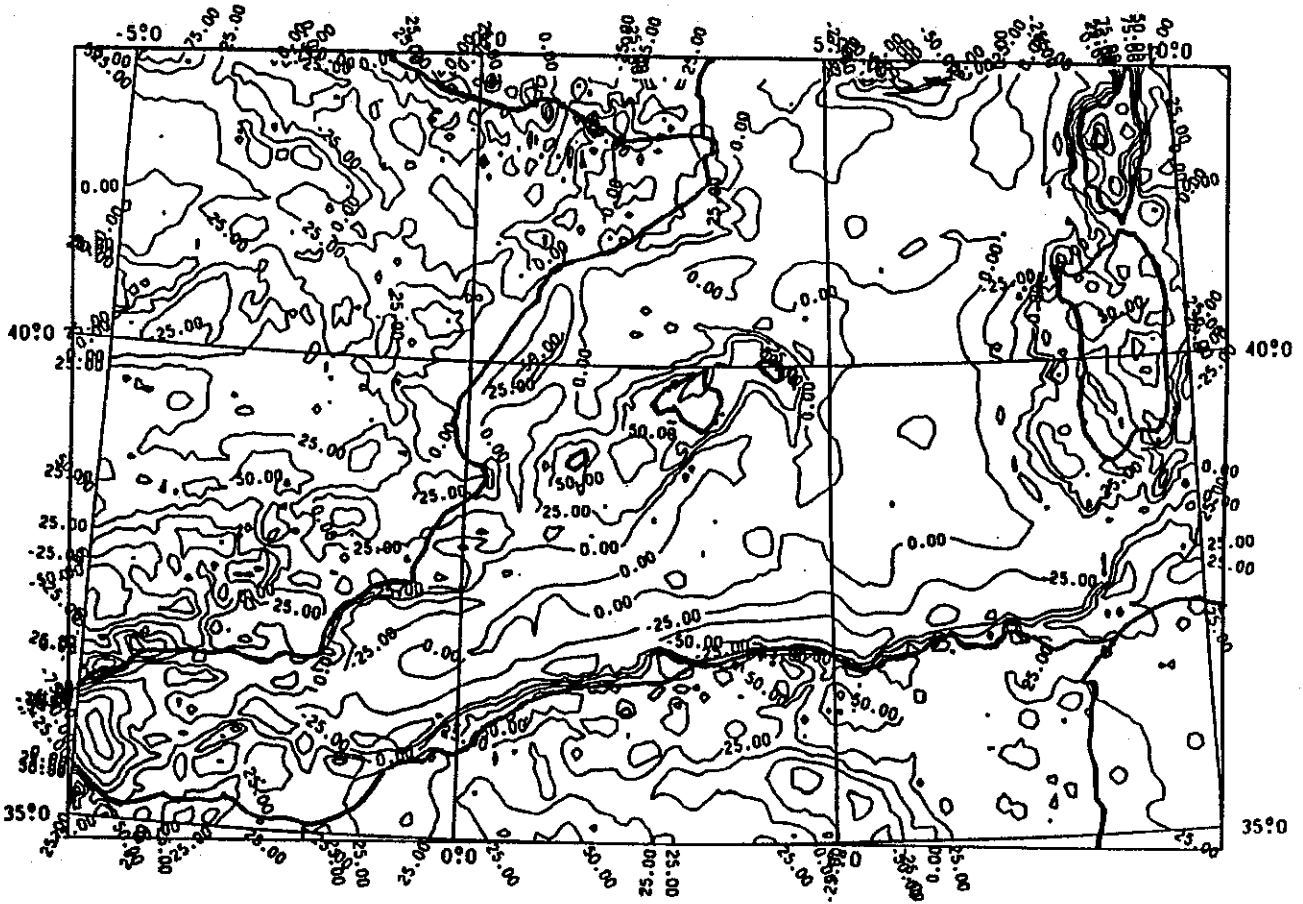


Fig. 1. - Improper reference surface in RTE computation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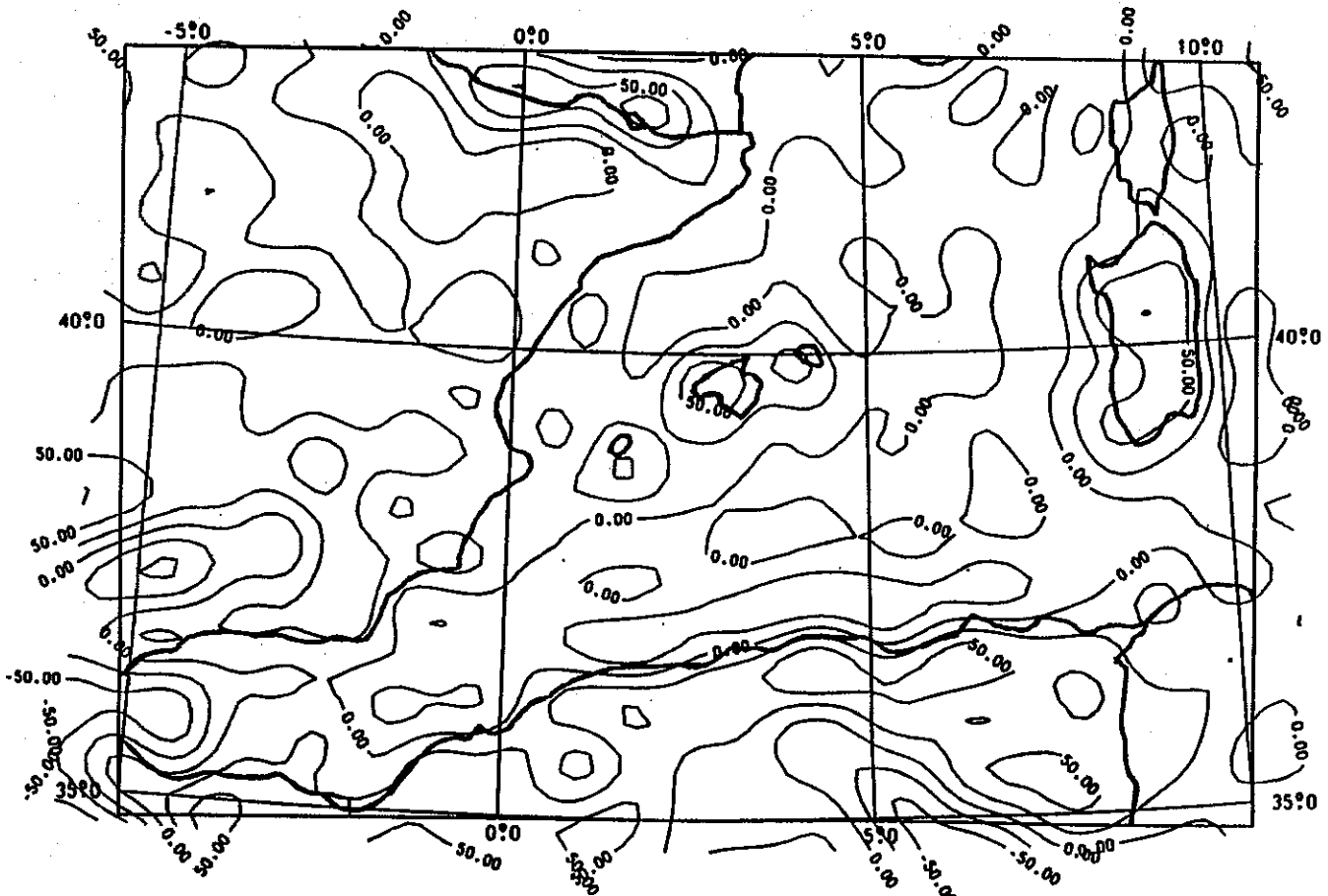
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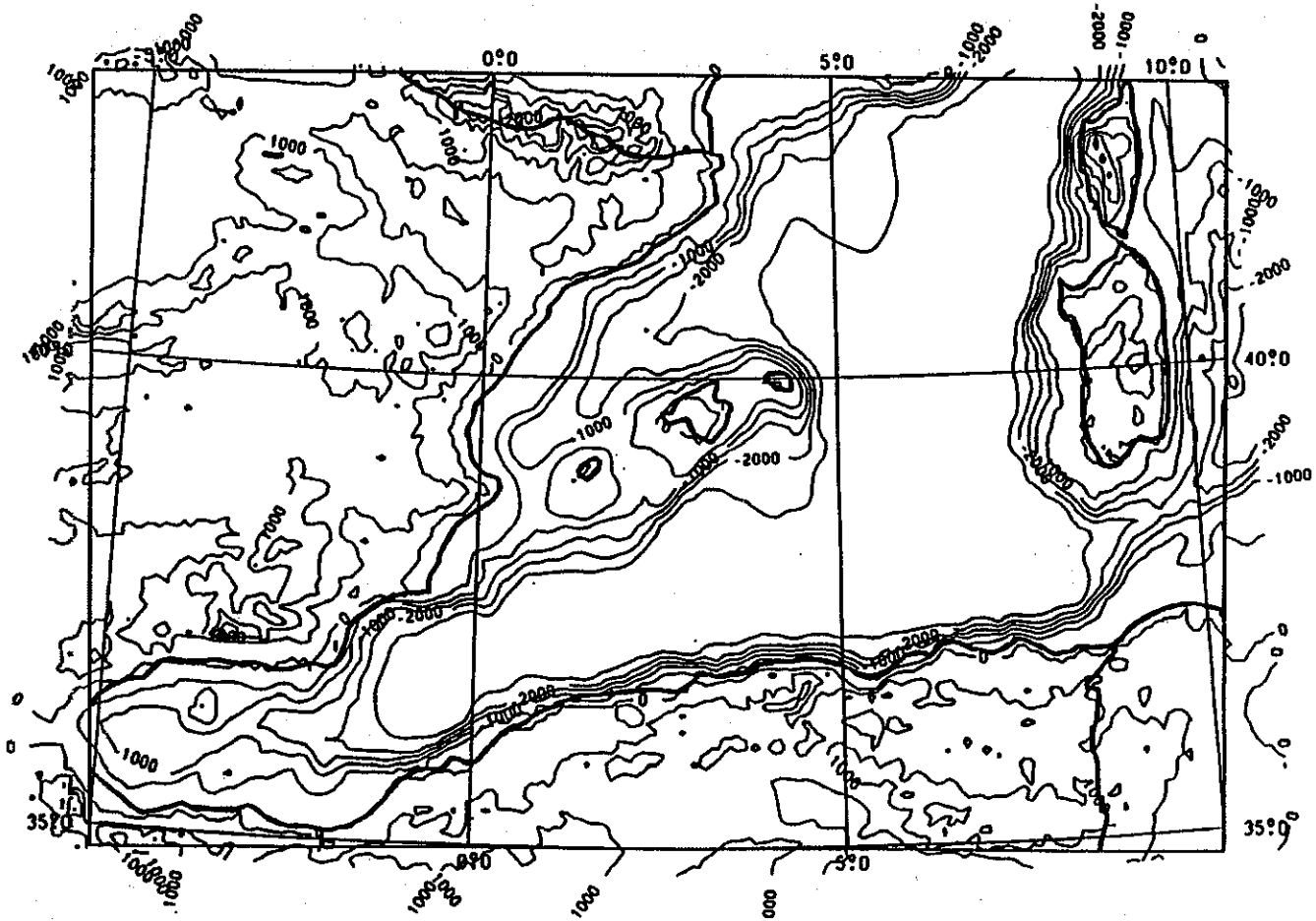
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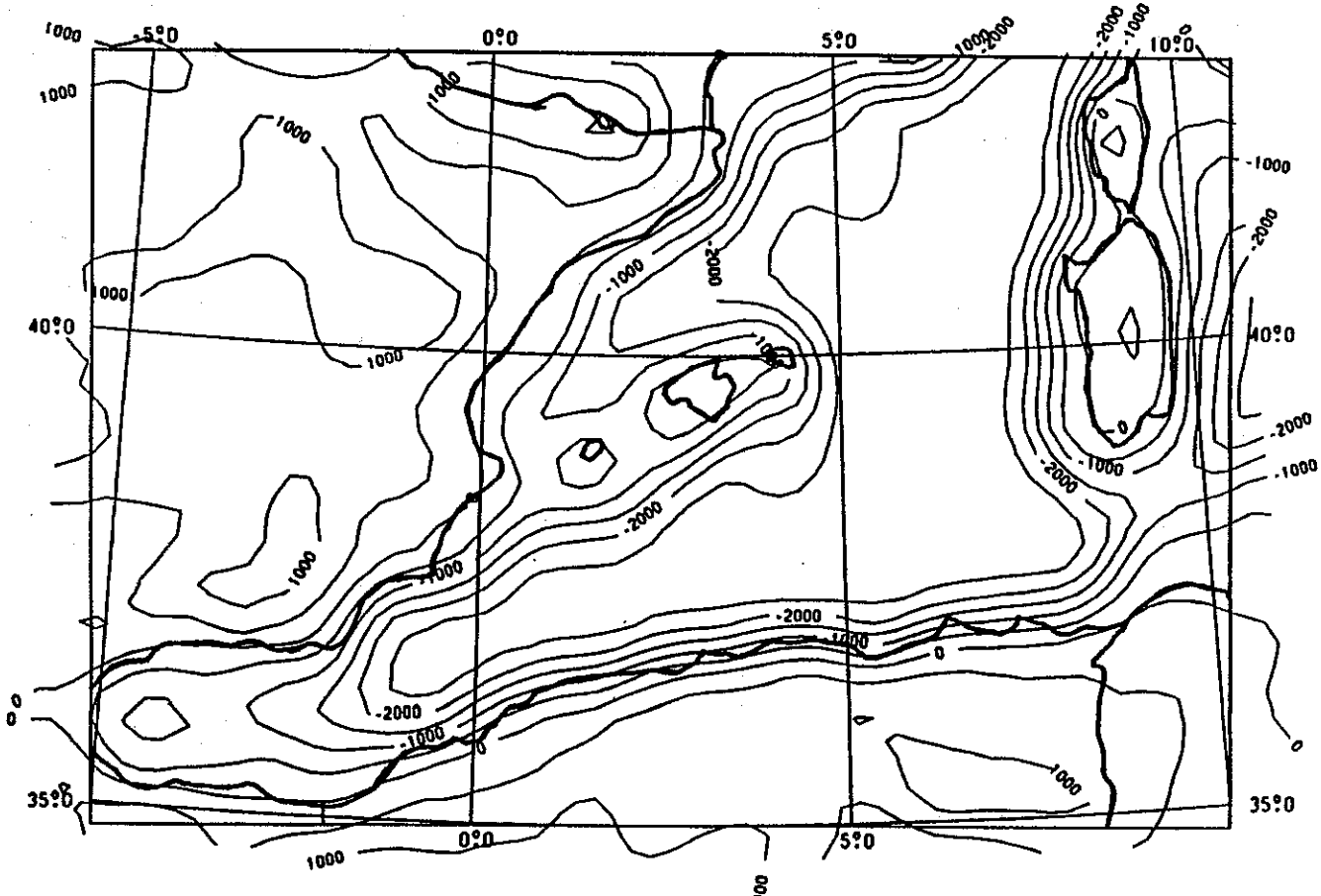
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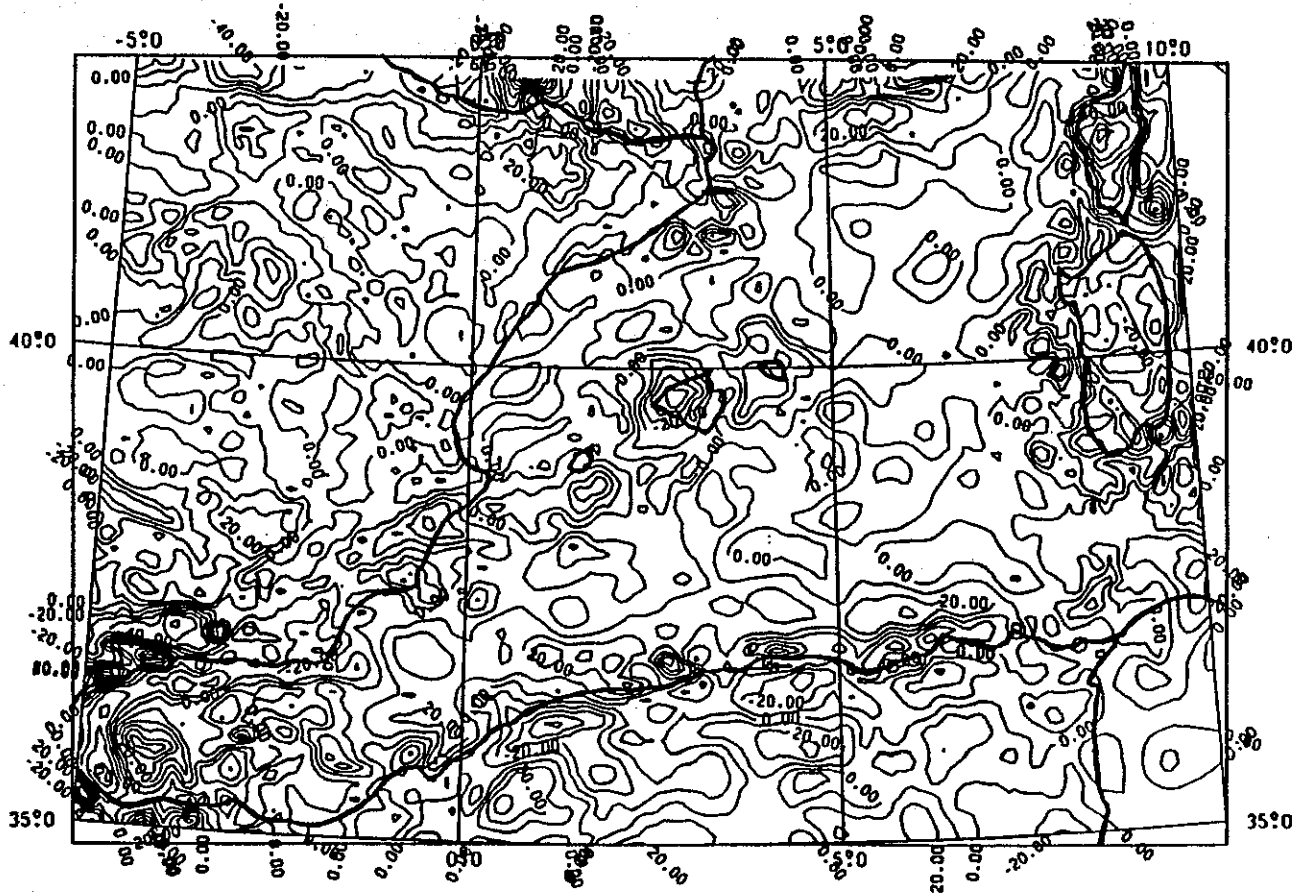
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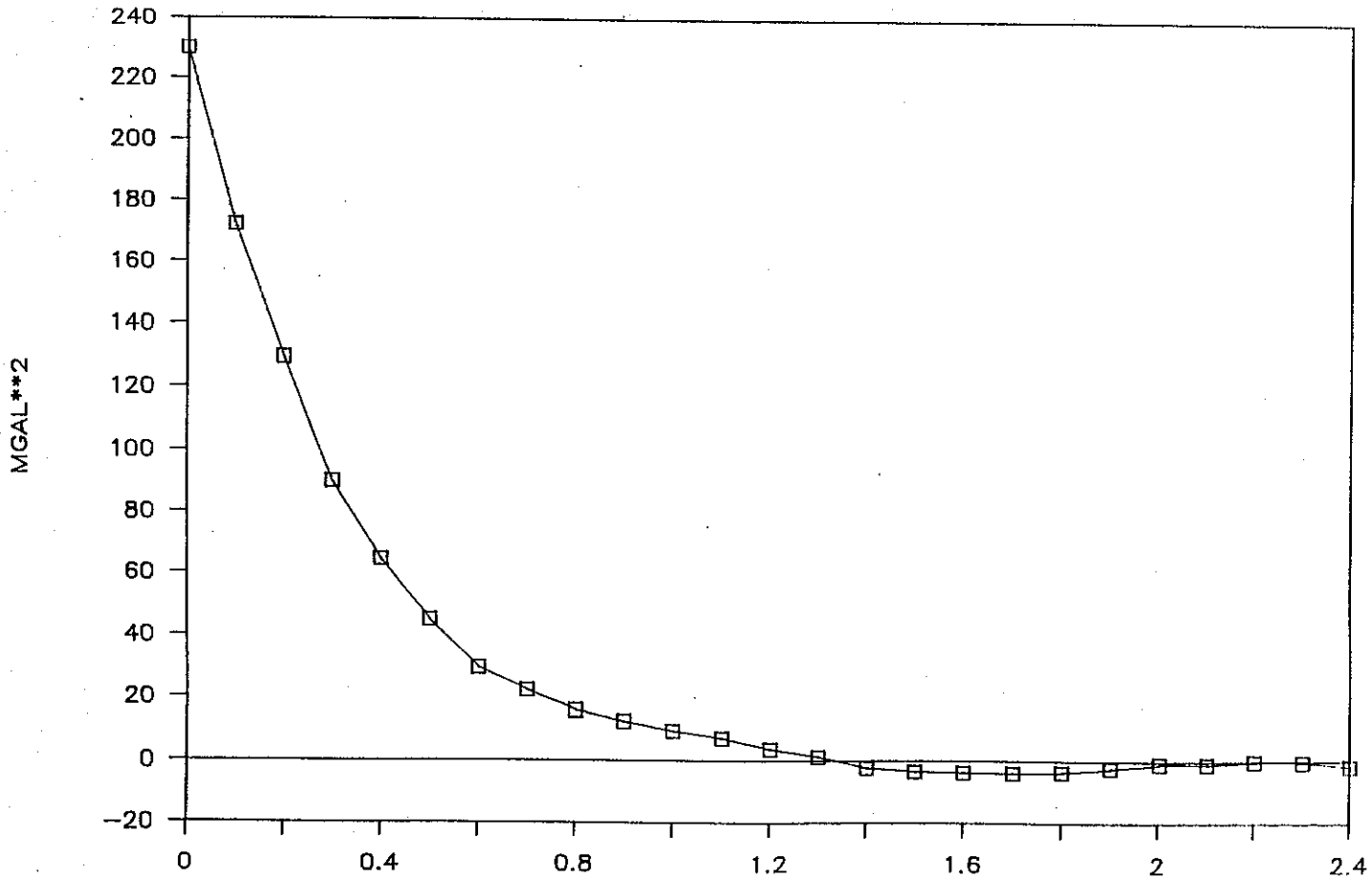
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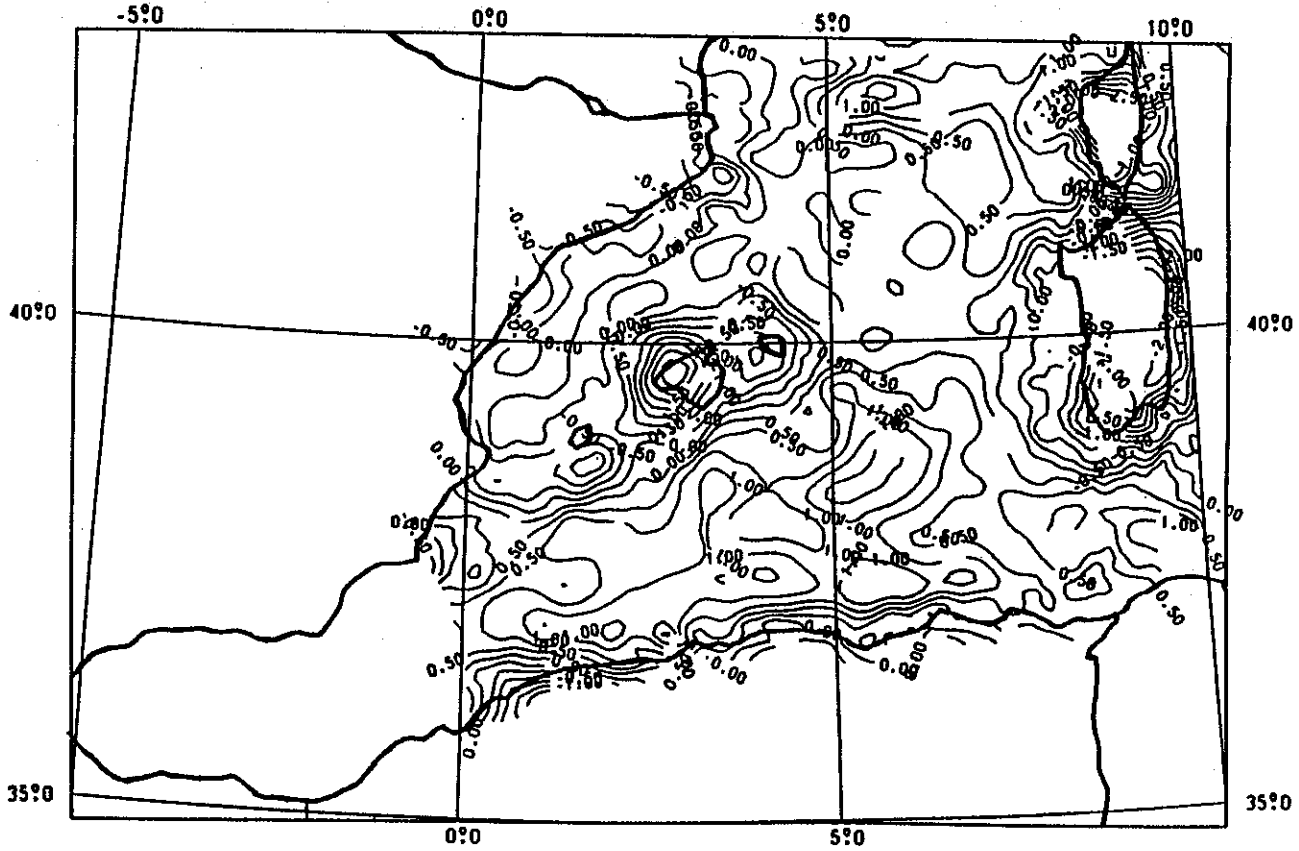
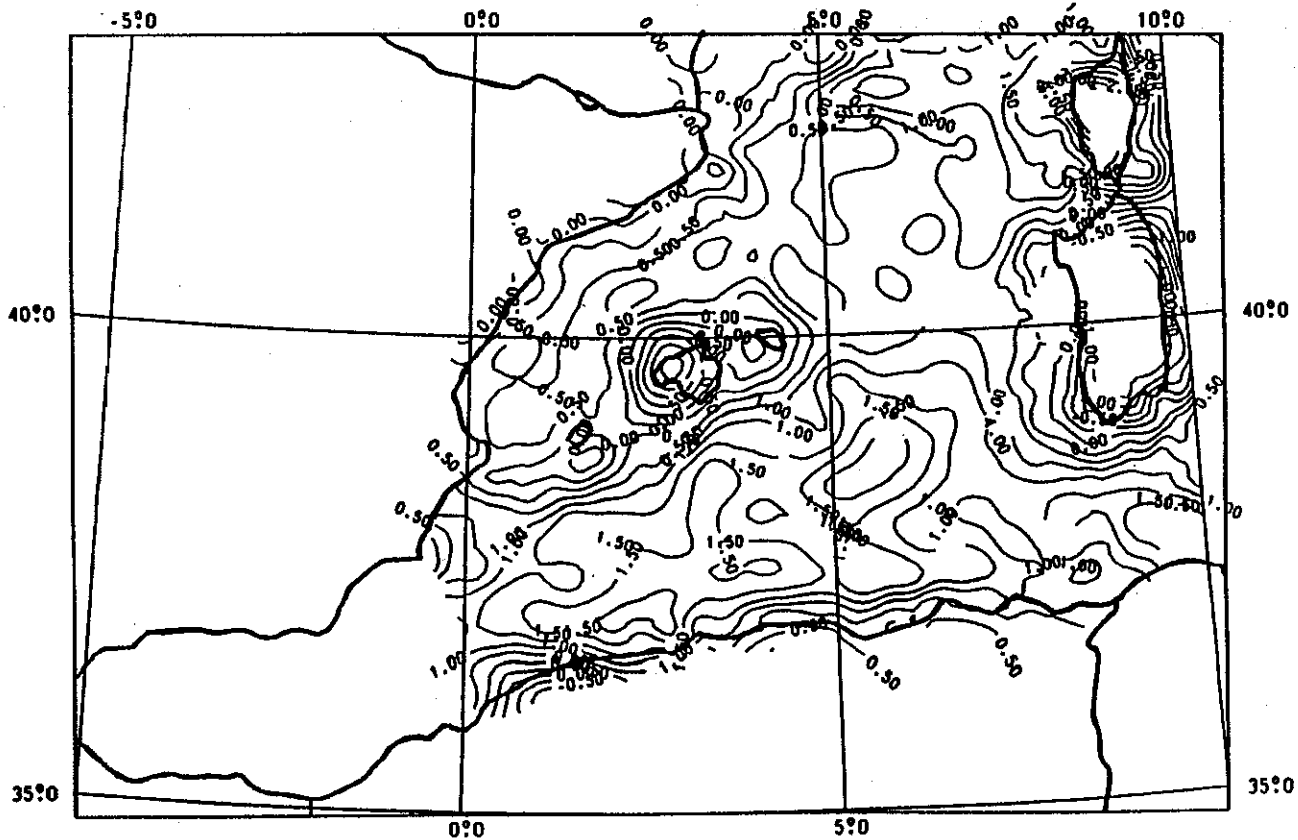


Fig. 8. - Stokes/FFT residual geoid (m)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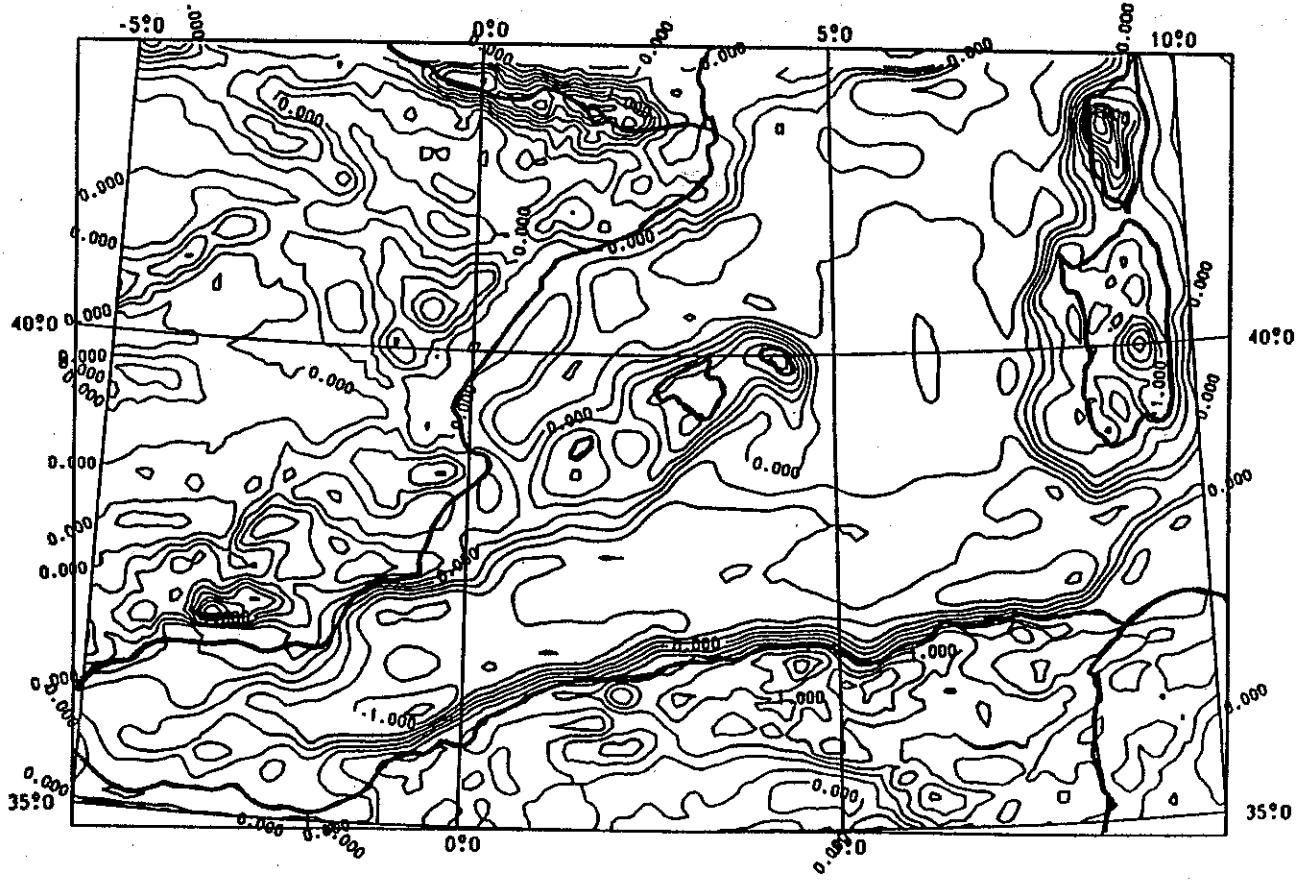
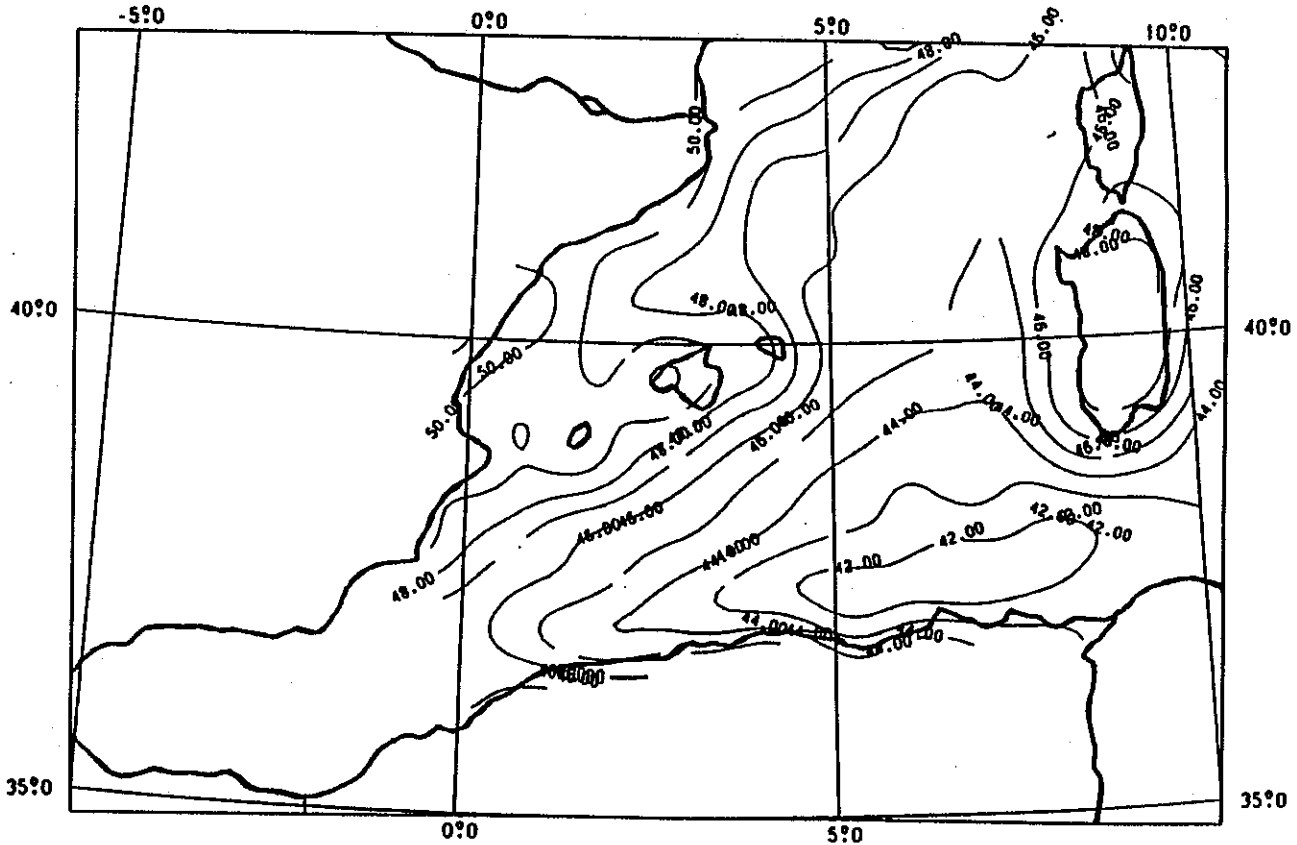


Fig. 10. - RTE geiod (m)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48.00
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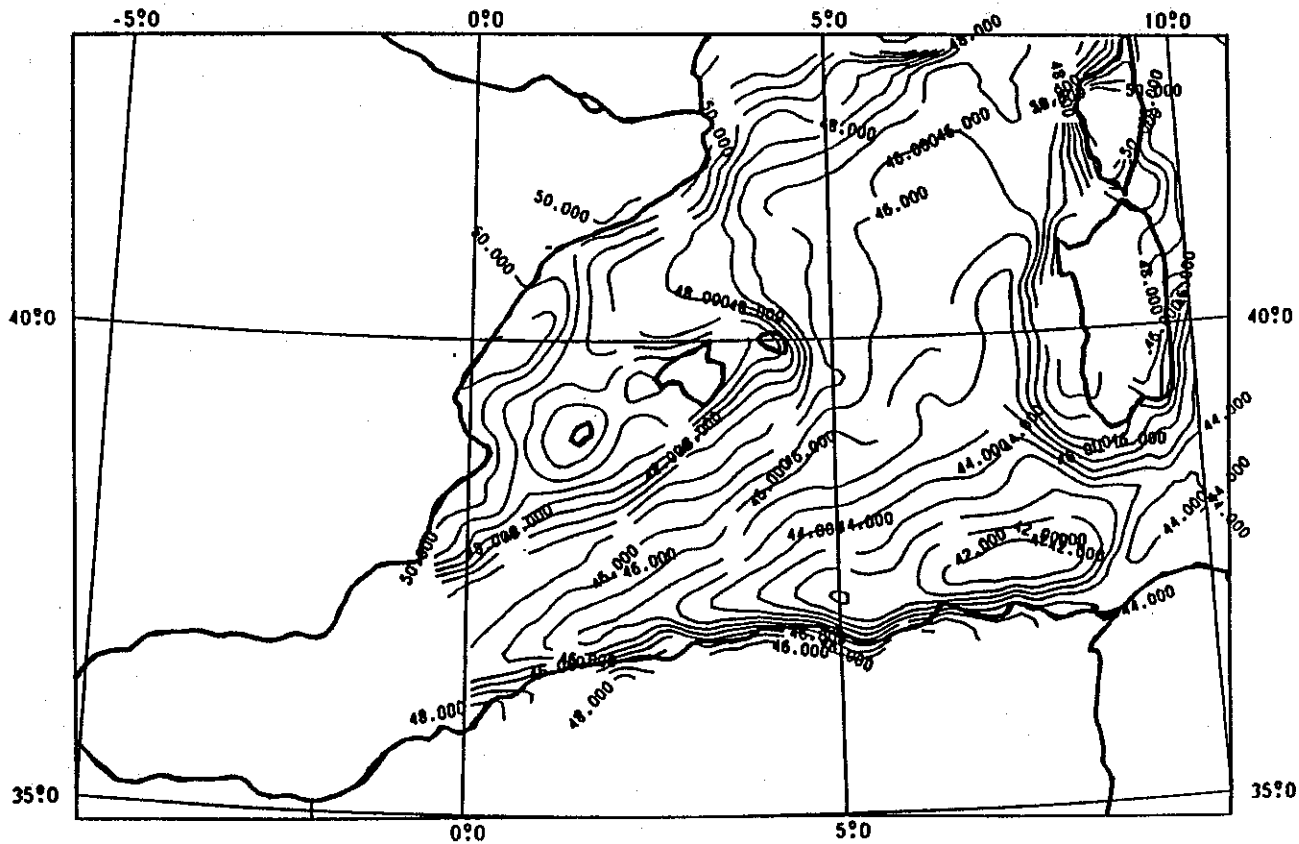
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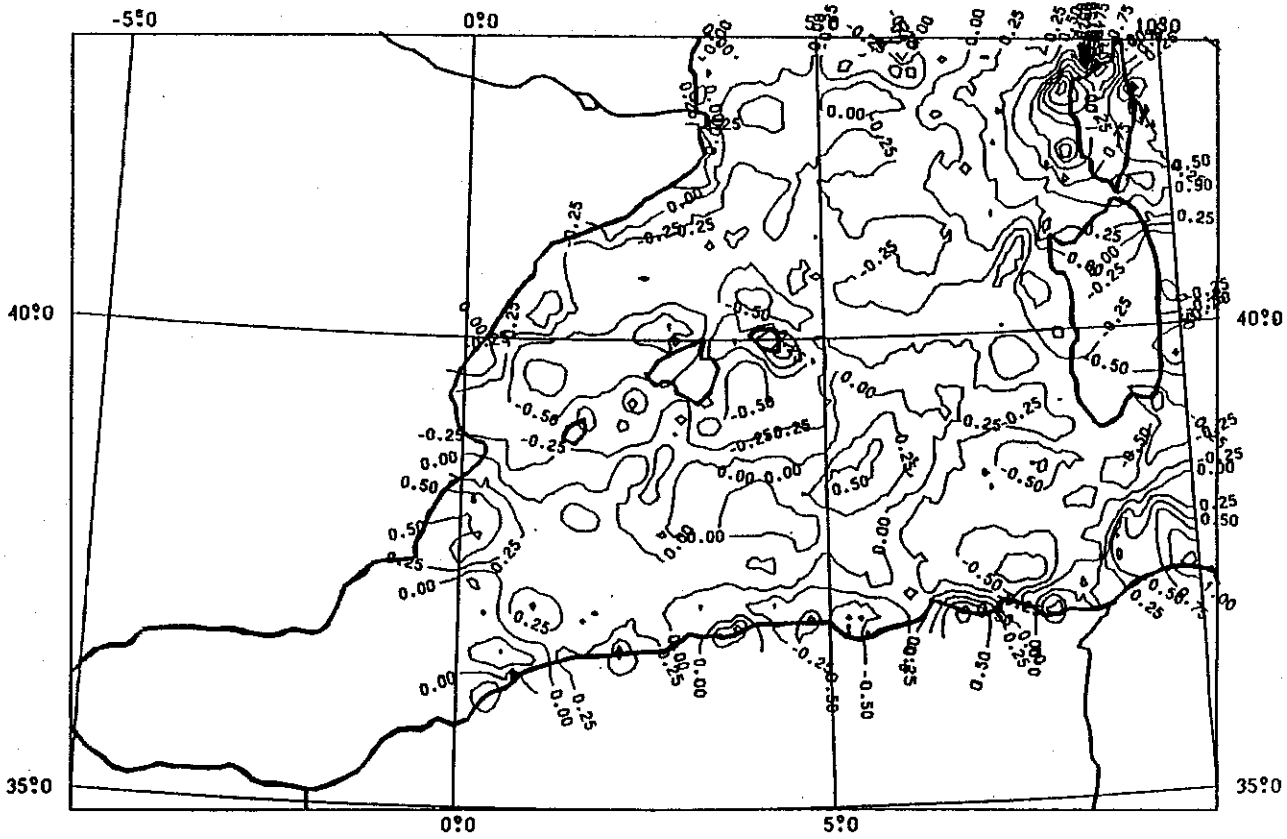
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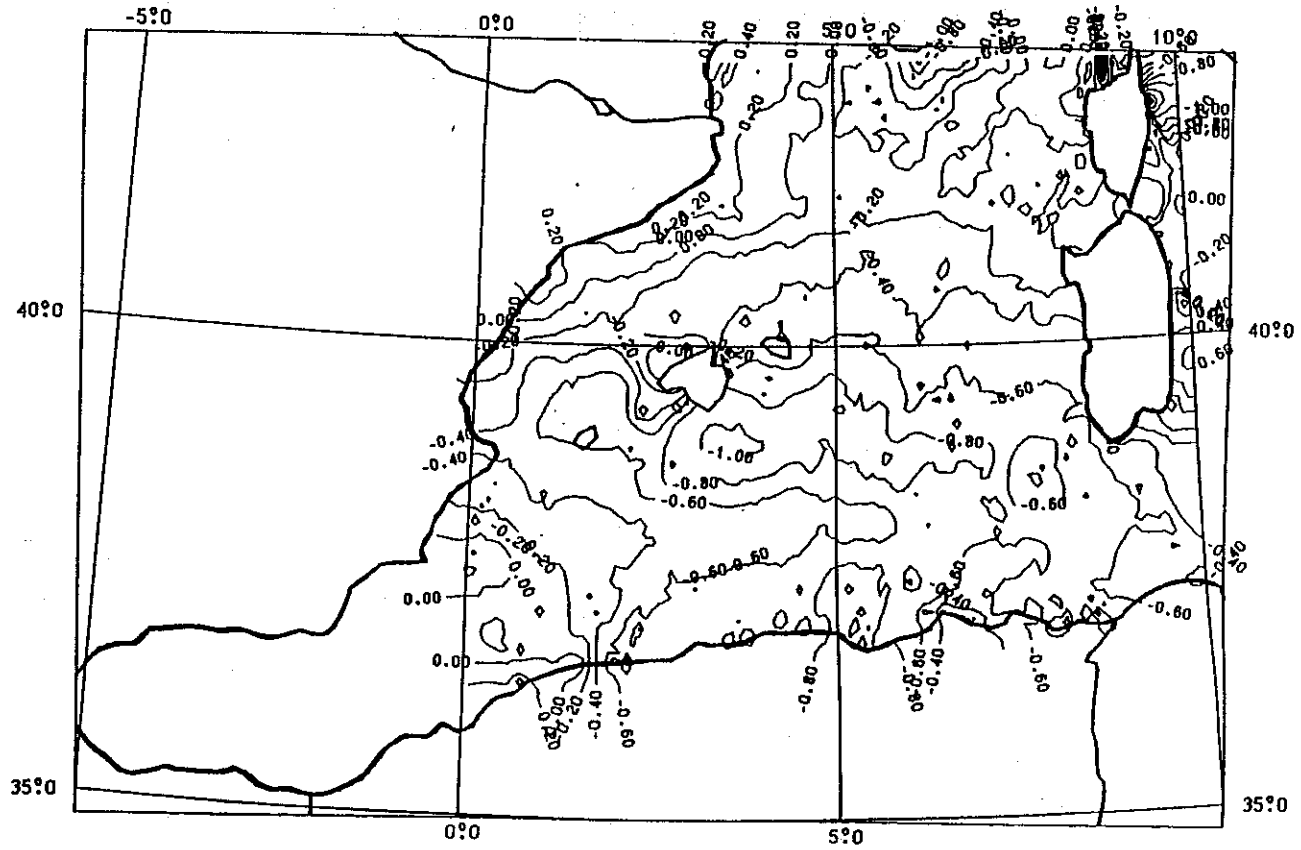

Fig. 14. - Sea Surface Topography (m)