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# Height Determination using GPS
# The need of a precise geoid.

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**Sumari:** Emprant el GPS es possible obtenir diferències d'alçada amb una precisió de 1 ppm on inclùs menys. Per a estacions que tenen considerables diferències en alçada, aquesta mesura però pot requerir que la refracció tropoesfèrica sigui considerada d'una manera especial.

Tant per raons pràctiques com cientifiques son emprades les alçades sobre el nivell promig del mar (alçades normals o ortomètriques). Això requereix la conversió de les diferències d'alcada el.lipsoidal en alcades MSL, fet, que al seu torn, precisa que les diferències geoïdals (o quasi geoïdals) siguin a l'abast.

Si hi ha suficients dades de gravetat, o bé es disposa de marques d'anivellament, llavors, la cota geoïdal pot ser calculada per diversos mètodes amb alta precisió. Una distribució dels punts de gravetat de 10 a 20 kilòmetres és en la majoria dels casos suficient per a obtenir una precisió comparable a la que s'obté emprant anivellament trigonomètric. En àrees de les quals no es disposa de dades de gravetat però, probablement es requerirà un espaiat més dens i un cobriment més ampli de l'àrea. Això és degut a que aquesta informació pot no estar inclosa en els models de camp de gravetat esfèrica i harmònica emprats com a base dels càlculs de la geoïdal.

**Abstract:** Using GPS it is possible to obtain ellipsoidal height differences with a precision of 1 ppm or better. For stations having large height differences this may require that the tropospheric refraction is accounted for in a non-standard manner.

For many practical and scientific purposes heights above mean sea level (normal or orthometric heights) are used. This requires the conversion of the ellipsoidal height differences into MSL heights. This again requires that geoid (or quasi-geoid) differences are made available.

If sufficient gravity data or levelling bench-marks are available, then the geoid may be computed by a number of methods with high precision. A gravity spacing of 10 - 20 km is in most cases sufficient to give a precision comparable to what is obtained using trigonometric levelling. In areas with no prior gravity surveys, however, a denser spacing and rather large areal coverage may be needed. This is because this information will not be included in the spherical harmonic gravity field models normally used as basis for the geoid computations.

## 1. Introduction.

The Global Positioning System is now nearly fully developed, and we see a rapid development of its use in navigation, surveying and high precision geodesy and geodynamics. Its best results are presented in the form of baseline distances, which are independent of the coordinate system origin and orientation used. It is well known that even when precise ephemeris are used, then the coordinate system origin (the Earth center of mass) varies 0.2 -0.5 m.

If we present the GPS survey results as coordinate differences (ellipsoidal latitude, $\phi$, longitude , $\lambda$, and height, h) then they will not be completely independent of the variation of the system origin, but for smaller distances the error will be limited. (We presuppose that precise ephmeris are used).

The ellipsoidal height is influences by tropospheric refraction. If the station separation is small, and the height difference is small, the effect cancels. But results from mountaneous areas which we discuss in section 2 show that the treatment of the refraction has to be given special attention in this case.

For most surveying applications, the height above mean sea level is needed. It may be readily obtained if we know and subtract the height of the geoid or the quasi-geoid from the ellipsoidal heights. Most commercial GPS processing packages include geoid information computed from recent global spherical harmonic expansions like the OSU91A solution (Rapp et al., 1991). These solutions may be sufficient for many purposes if used in areas where the gravity information in the area has been used to construct the model. On the other hand, it is possible with a little extra effort to make local improvements on a global solution, so that the resulting heights are as good as these obtained e.g. from trigonometric levelling. We discuss the need for geoid information in section 3 and 4, and the need for additional data, if improvements are needed in section 5.

While the national surveys of most developed countries now provide (sell) geoid information to GPS users, this information is still not available with sufficient accuracy in many areas. However methods for computation are available and implemented as FORTRAN programs. In section 6 a brief outline is given of the methods available, and hints about where help may be obtained are given.

## 2. Determination of ellipsoidal heights.

Using GPS we are able to obtain precise position differences. Hence if we need the ellipsoidal height, a Satellite Laser or VLBI site must be included in the survey. In Europe we may in the future be able to use one of the new EUREF-sites, see Seeger et al.(1990).

The quality of the results depends on the distance between the observation points, the altitude difference and the availability of precise orbit information. (The instrument type naturally also plays a role, but this is not the issue here.)

The position difference may be given either as a difference in Cartesian (X,Y,Z) coordinates ~~or as~~ difference in geodetic latitude, longitude and ellipsoidal height. The height difference $\Delta h = h_1 - h_2$ between two points $P_1$ and $P_2$ is often of slightly lower quality than the horizontal position differences. (We miss being able to observe satellites under the Earth surface, on the other side of the Earth).

Especially for points with large height differences (larger than 500 m), the corrections due to the troposphere plays an important role (Hollmann & Welsch, 1990, Gurtner et al. 1987). Different methods for modelling the delay due to tropospheric refraction may cause height difference variations of up to $2*10^{-4}$. It may even be dangerous to use meteorological information collected at the observation site, if the values are not representative for the surrounding atmosphere.

However, under reasonably good conditions, $\Delta h$ may be determined using static positioning within a few cm for station separation up to 50 km. Longer distances require as mentioned above the use of stations with precisely known geocentric coordinates.

Kinematic positioning gives nearly equally good height differences provided dual frequency receivers are used, see e.g. Baustert et al. (1990).

**3. The use of heights above mean sea level (MSL).**

The ellipsoidal height differences contain essential information which without further problems may be used when monitoring recent height changes (Smit, 1991, Genrich & Boch, 1992), or in photogrammetric applications. But the height concept used in practice is of physical origin, the height above mean sea level. Remember also, that in precise levelling, potential differences, and not geometric height differences are used.

There are two main systems of heights above MSL, the orthometric height, H, and the normal height, $H^*$, cf. Figure 1. Conceptually they are different, and their difference is proportional to the height difference multiplied with the Bouguer gravity anomaly.

The orthometric height is the height above the geoid, measured along the plumb line. Since this line generally pass through the topographic masses, its computation is somewhat difficult. The normal height of a point P is the distance along the ellipsoidal normal from the ellipsoid to a point Q which has the normal potential **U** equal to the potential **W** of the point P.

The difference $N = h - H$ is the geoid height, and the difference $\zeta = h - H^*$ is the height anomaly. (At the ocean surface the two quantities are identical). Hence, if we know the geoid height or the height anomaly, then we are able to compute any of the two heights above MSL. In the following we will only deal with the normal height, since we from this may compute the orthometric height if needed. (Supplemental data, like gravity values, may be needed).

The difference between the gravity potential of the Earth and the normal potential, $T = W - U$, is called the anomalous potential. If we know T, then using Bruns formula,

$$\zeta = T/\gamma \quad (1)$$

Here $\gamma$ is the normal gravity (the magnitude of the gradient of U).

If we subtract the centrifugal part of the potential W, then it is a harmonic function, which may be expressed using spherical harmonic functions. Such functions including more than 130000 terms have been computed primarily at the Ohio State University, see Rapp et al. (1991). Since U is a simple function, T may be computed by subtraction, and using eq.(1) we find the height anomaly. This is the basis for the calculated heights above MSL provided in the software packages by most instrument manufacturers. In practice the height anomaly at zero altitude (nearly identical to the geoid height) is tabulated, and the table is used for interpolation. The surface formed by the points having a distance from the ellipsoid equal to the height anomaly is called the quasi-geoid. In the following we will not distinguish between the geoid and the quasi-geoid.

## 4. How well do we need to know the geoid ?

For many applications the geoid heights computed from spherical harmonic expansions like the OSU91A field are sufficient. In many cases heights above MSL may be obtained with an average error lower than 0.5 m. However, one have to be careful, because such fields are only good in areas where data have been available for their calculation, like in most of Western Europe, including Spain. But in Asia, Antarctica and Greenland (Forsberg et al. 1992) the quality is low, 2.0 - 3.0 m.

The spherical harmonic expansions may fortunately be improved locally or regionally. The most strict requirement to an improvement must be that we are able to calculate the MSL height differences with the same precision as the GPS derived ellipsoidal height differences. This means a few cm over 100 km distance.

This is easily obtained, if sufficient gravity data is available in the region where the GPS survey is executed. The main difficulty arises in mountains, where we as mentioned in section 2 also may have difficulties using GPS.

We must distinguish between various applications in order to specify the precision requirements to the geoid height differences. For mapping applications a precision like the one obtainable from trigonometric levelling, i.e. 5 - 10 cm over 50 km should be sufficient. For engineering surveys which has the purpose of monitoring large constructions and especially in geodynamic applications sub centimeter precision may be needed. However, here the use of the geoid is only relevant, if older precise levelling results have to be used in the control (Madsen & Tscherning, 1989), or if in fact a level surface has to be identified, see e.g. Leick et al. (1992).

Another area where a high precision is needed is the monitoring of the surfaces of ice caps like the Greenland ice sheet (Gundestrup et al. 1986, Forsberg et al. 1992). Here height differences good to 0.25 0.5 m over distances of 1000 km - 2000 km are needed, corresponding to precision better than 1 ppm.

## 5. Estimation of amount of auxiliary data needed to obtain a precise geoid.

We will now limit ourselves to land areas. (GPS may also be used for monitoring the sea surface topography variations, see e.g. Hein et al. (1990)).

On land we will in developed countries generally be operating where there exist some bench marks with height information, i.e. we know here the height above MSL. If we observe with GPS in these points, ellipsoidal height differences will be obtained, and we may then calculate the corresponding geoid height differences. If we from these differences subtract the values calculated from e.g. the OSU91A model we will be left with small residuals suitable for interpolation. After the interpolation, the contribution from the model will have to be added back. This is called a remove-restore method.

If the residuals are not smooth, then it is typically due to the attraction of the topographic masses. The potential of this attraction may easily be calculated if a DTM is available, and its contribution to the geoid is obtained using eq. (1). Its removal will again smooth the geoid residuals (new residuals are obtained), and after the interpolation, the potential must be added back in order to get the total geoid. Unfortunately this procedure seems not yet to have entered into practice, possibly because the location of many bench marks is not suitable for GPS observations.

If very few bench marks are available, the geoid has to be calculated using gravity data. So the question is how dense a gravity coverage is needed, and which size of surrounding area should be covered. The answer to the last question is related to the quality of the spherical harmonic expansion used as base for the remove-restore process. As a rule of thumb, the area should at least have an extend out to a distance equal to 180° divided by the degree and order of the spherical harmonic expansion. For the OSU91A model the maximal degree is 360, so the distance should be 55 km.

The data spacing depends on the variability of the residual gravity field. If it varies due to the presence of disturbing topographic masses, then their effect should be removed and restored as discussed above. If this is done, all residual gravity fields look quite a lot the same. (We make everything look like the smooth gravity field of Denmark). The variation may be expressed using a function expressing the correlation between two (residual) gravity values as a function of the distance between the points. The correlation will obviously be maximal (100 %) at zero distance, and decrease rapidly so that it becomes 50 % at a distance between 5 and 10 km. Gravity values becomes then uncorrelated at about 25 km distance. If the correlations are multiplied with the gravity variance, then covariances are obtained. Fig. 2 show the residual gravity anomaly covariance function of Cataluna, cf. Andreu & Simo (1990).

Using the covariance function the geoid may be determined using an optimal estimation method called least-squares collocation (Moritz, 1980, Tscherning, 1985). This method also provides error estimates, which may be calculated for various data spacings and areal coverage.

In areas where the spherical harmonic expansion is of good quality (i.e. in areas where the gravity field has been surveyed) we typically have a variation of the residual gravity of ±15 mgal ($10^{-5}$ $m/s^2$) and a geoid variation of ±0.5 m. In areas with lacking gravity surveys (e.g. Greenland) the gravity variation is typically ±35 mgal and the geoid variation ±2 m.

Results of simulations are found in Forsberg & Madsen (1990, Table 2) for a case where the spherical harmonic model is good, and in Tscherning (1983) for a situation where it is "bad". For the first case it is shown that an error less than 1 ppm is obtained using a gravity spacing of just 10' (22 km). This means that for a local GPS survey covering an area of for example 50 x 50 km, we need gravity data in the area 55 km outside this area. This means that about 250 gravity observations must be made or obtained from the national gravity databases. If data has to be observed it is a good idea to carry along a gravity meter which then may be read while the GPS instruments collects data. If data are to be observed outside the area where the GPS survey takes place, then the coordinates of the gravity points may be obtained using kinematic GPS. A horizontal position of 25 m and a height good to 1 m is sufficient. (The data obtained in this manner will be gravity disturbances and not gravity anomalies).

For the simulation executed in the case where there did not exist sufficient gravity information to assure a good quality of the spherical harmonic expansion, a 10' gravity spacing gives a 5 ppm error of the (residual) geoid height differences. The data collection area also has to be large, and the effort to collect the necessary data may not be compatible with the task of executing the GPS survey.

## 6. Methods for geoid determination.

Several methods are available for geoid determination from gravity data. They all give approximatively the same result if sufficient well distributed data is available. For all methods the remove-restore technique may be applied. However,some methods require that the data first are "downward continued" to zero altitude. This causes a small complication which we will not deal with here.

In principle the computation requires that the data are available globally. But here the removal of a spherical harmonic expansion tends to remove (or represent) the contribution from data at a distance. This permit the use of a limited data collection distance as discussed above.

The most severe problem occurs if data are missing, e.g. due to the occurrence of a lake or shallow waters which (at least earlier) limited the possibility for carrying out a sea-gravity survey. Here supplemental gravity field information like deflections of the vertical may aid in bridging the data gap.

The above mentioned method of least squares collocation will function even if there are data gaps, and it permits the combination of data of various kinds. The areas where the result is of low quality are easily found by calculating the error estimates for the whole area of interest.

The method is quite demanding of computer resources, because the use of the method involves the solution of a system of equations with as many unknowns (typically 500 -1000) as the number of observations. The method has therefore primarily been used for smaller areas, or a larger area has been covered by overlapping solutions. The method has been successfully used in Cataluna (Andreu & Simo, 1990), parts of UK (Gerrard, 1991), Greece (Arabelos, 1980), Italy (Benciolini et al. 1991), the Nordic area (Tscherning & Forsberg, 1986), Turkey (Amin, 1991) and parts of Germany (Denker, 1988), Austria (Suenkel, 1983). Spain has been included in a solution for the whole Mediterranean (Sevilla et al. 1991).

Fast, but less flexible methods, are based on the Stokes formula (Moritz, 1980). The origin of the method is the simple spectral relationship between gravity and the geoid, which primarily is a multiplication with a factor proportional with the frequency. This relationship has enabled the use of the Fourier method, and computationally the use of the Fast Fourier Transform (FFT) (Nagy & Fury, 1990, Schwarz et al.,1990). The success of the method is closely related to the use of the remove-restore technique, which reduces the errors associated with the use of a planar approximation.

The FFT method has been used recently for large areas like the US (Milbert, 1991), the Nordic area (Forsberg, 1990) and the Eastern Mediterranean (Arabelos et al. 1991).

The Stokes formula may also be used directly. It may be modified in order to take into account that residual gravity anomalies are used, see Sjoeberg (1986).

A software package **GRAVSOFT** (Tscherning et al. 1991) is available commercially from the National Survey and Cadastre - Denmark or the Geophysical Institute. Versions which may run on a PC are also available. (The package is distributed for a limited cost if used only for research).

## 7. GPS tests of geoids.

One may then ask, whether the results claimed above really hold in practice. Here independent controls have been made, where the geoid has been computed from gravity, and then compared with values obtained by determining ellipsoidal height differences in precise levelling points see Torge et al. (1990). The result presented in this publication is remarkable because it shows an agrement in the decimeter range for a nearly 3000 km long GPS traverse. Other interesting results are found in (Mainville et al.1990, Sideris and Forsberg, 1991), see also the papers in Rapp & Sanso (1991).

Controls have also been made in smaller areas, giving agreements at the cm level, see e.g. Denker & Wenzel (1987), Dodson & Gerrard (1990), Engelis et al. (1984, 1985), Madsen & Aarestrup (1990), Leick et al. (1992), Schwarz et al.(1987).

The geoids may also be tested in areas without precise levelling, e.g. as done in Cataluna (I.Colomina, private information, 1992), using deflections of the vertical. Along the coast satellite altimetry may also be used as a control. Here may however be problems due to sea surface topography variations caused by temperature and salinity differences.

## 8. Conclusion.

GPS has been used successfully for the determination of ellipsoidal height differences. In areas where gravity data are available, they may easily be converted to heights above MSL. This requires that a precise geoid is available.

The task of computing a geoid is not difficult, but requires a good background in physical geodesy, which one does not find usually in the surveying community. The task also requires international cooperation, since data from surrounding areas are needed. It should be the obligation of the national agencies to provide (contingently on a commercial basis) the necessary precise geoid information, and this obligation is also already being met in many countries or independent regions.

Hopefully the International Geoid Service will soon be functioning based at the Politecnico de Milano, and supervised by the International Geoid Commission of the International Association of Geodesy. This should be the place, where training in geoid computation could be carried out, and where an expertise is found which may advise on geoid computation.

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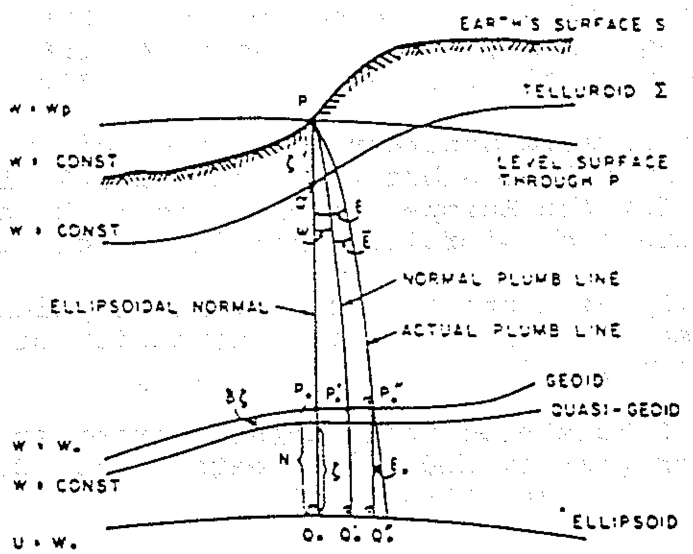


Figure 1. The geometry of the classical BVP and the Molodensky BVP.

Figure 2. Covariance functions of (1) free–air gravity, (2) free–air gravity minus contribution from spherical harmonic expansion to degree 360 and (3) as (2), but also residual topographic contribution subtracted.