A STRATEGY FOR GROSS-ERROR DETECTION IN SATELLITE ALTIMETER DATA APPLIED IN THE BALTIC-SEA AREA FOR ENHANCED GEOID AND GRAVITY DETERMINATION

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Abstract:

Satellite altimeter data has successfully been used for gravity field modelling after global or local cross-over adjustment. However, especially in coastal areas are the data contaminated by gross-errors and by biasses and tilts which have not been eliminated due to the missing cross-over with other tracks.

Using least-squares collocation (LSC) with altimeter track biases and tilts as additional unknowns it is possible to compute adjusted altimeter heights. A comparison of the difference between the original and the adjusted value with the error estimate obtained from LSC may then be used to find suspected gross-errors.

The procedure was used in two areas in the Baltic Sea with and without taking advantage of high quality gravity data in addition to a spherical harmonic expansion (IFE88E2) complete to degree 360.

Gross-errors were found in 1 % (12) of the 1232 values used. The additional use of gravity values reveiled only a few errors (4) in addition to these already detected using a 3-sigma level.

However, the additional use of point gravity data is significant in improving the power of the method. This is seen from the estimated errors of the track biases, which were 0.15 m for connected and 0.22 m for un-connected tracks respectively when no gravity data were used. When gravity data were included the error estimates decreased to 0.08 m and 0.13 m, respectively. A similar improvement in the geoid was also found, while the estimated error of the predicted gravity values in areas with only altimetry decreased from 5 to 4 mgal, typically.

Draft prepared for the XI meeting of the Nordic Geodetic Commission, Copenhagen, May, 1990 and the Geoid Commission Symposium, Milan, June, 1990.

1. Introduction

Satellite altimeter data has successfully been applied for gravity field modelling after global or local cross-over adjustment see e.g. Rowlands (1981), Engelis and Knudsen (1989), Knudsen et al. (1988). However, especially in coastal areas are the data contaminated by gross-errors or by biasses and tilts which have not been eliminated due to missing cross-over with other tracks.

It is important to be able to carry out precise gravity field modelling in coastal areas, so the above mentioned difficulties must be overcome. The problem of missing cross-overs has been overcome, see Tscherning and Knudsen (1986), and the elimination of gross-errors will be the main subject of this paper.

Coastal areas are important, because we here usually have small depths, which makes it feasible to carry out exploration for various types of ressources. Consequently there frequently exist gravity data here, or at the coast, which we should be able to take advantage of when scarching for gross-errors or estimating biasses or tilts.

In section 2 we will discuss the applicability of some of the methods usually applied for gross-error detection in gravity field data. One of the methods, least-squares collocation adapted to this purpose, does also permit the use of different types of data, and this is discussed in section 3. We have used the method in two local areas in the Baltic Sea, and the results obtained here are found in section 4.

2. Gross-error detection in gravity field data.

It is well known that gravity field data are spatially correlated. Gravity field quantities of the same type and not far apart will be very similar. Data from a local area from which a regional trend, and local irregularities due to the attraction of topographic masses has been subtracted will (in most cases) have a nice symmetric distribution. In many cases it will even be close to a normal distribution.

A simple formation of a histogram for the data in an area will therefore show outliers as data not belonging to the distribution of the main part of the data. Data closely located may also be compared, and if a value is very different from its neighbour, it may be regarded an outlier. 2-D contouring of the data is here a very efficient tool, see Bureau Gravimetrique International (1989).

With satellite altimeter data we may use these tools, but difficulties occur:

(1) due to biases or tilts will data from different tracks not have the same (zero) mean value

(2) data is not distributed evenly, so 2-D contouring is not always possible

(3) many errors occur at the end of a track or where it is broken by islands, so that some data have no or only one neighbour

(4) a large amount of data has to be evaluated in the future with new satellite missions (ERS-1, Topex-Poseidon), so it is not possible to visually inspect histograms or contour plots.

So an automatic, or semi-automatic, computer based method is needed. For gravity data local interpotation using least squares collocation (Moritz, 1980), has been used successfully for gross-error detection (B.G.I., 1989). A gravity anomaly, y, is predicted from a set of values, x_i, i = 1, ..., n, in neighbouring points, spaced as evenly as possible in all directions,

$$\tilde{y} = C_y \bar{C}^{-1} x \qquad (1)$$

Here C_y is the vector of covariance between y and the x_i values, $\bar{C} = C + D$ is the sum of the covariance matrix of the x_i quantities and the variance-covariance matrix of the noise (error) associated with the quantities. An error estimate may also be computed for the difference $|y-\tilde{y}|$,

$$\sigma^2(y-\tilde{y}) = C_o - C_y^T \, C^{-1} C_y \qquad (2)$$

where C_o is the variance of the gravity values. A gross-error is then detected, if

$$|y_{obs}-\tilde{y}| > k \, (\sigma^2(y-\tilde{y}) + \sigma_y^{\,2})^{1/2} \qquad (3)$$

where k is a constant generally having the value 3, and $\sigma_y^{\,2}$ is the error variance of the observation y_{obs}.

From eq. (2) and (3) we see an important fact. Gross-errors are most easily found if C_o is as small as possible. (3) is therefore first applied when the data have been smoothed by removing long-wavelength trends and local gravity field irregularities, e.g. caused by dominant topographic features.

The use of the method requires that the covariance function is known. The determination of this function using gravity data is discussed in e.g. Goad et al. (1984). It is simply estimated by forming mean values of products of observations having a spherical distance within a given interval. The same procedure can be used with altimeter data, but here the biases or tilts must be eliminated first using a cross-over adjustment.

3. Gross-error dectection using least squares collocation with parameters.

Suppose altimeter (or gravity observations) x_i, $i = 1, \ldots , n$ are expressed as

$$x_i = L_i(T) + A_i X + e_i \qquad (1)$$

where L_i is a linear functional applied on the anomalous (or "residual") potential T, X a vector of bias (and contingently tilt) parameters, A_i is a vector and e_i is the error. With altimeter or gravity observations the elements of A_i will be equal to 1 or 0 for bias parameters and time or distance difference for a track segment for tilt parameters. We will generally use a potential T obtained as the difference between the true potential and a spherical harmonic expansion and the attraction of the topography.

An estimate of a quantity $y = L(T)$, where L is a linear functional is then obtained by

$$\tilde{y} = C_y^T \bar{C}^{-1} (x - A\tilde{X}) \qquad (4)$$

$$\tilde{X} = (A^T \bar{C}^{-1} A + P)^{-1} A^T \bar{C}^{-1} x \qquad (5)$$

where P^{-1} is the a-priori variance-covariance matrix of the parameters (P mostly equal to zero). The error estimates becomes

$$\sigma^2(y - \tilde{y}) = C_o - C_y^{-1} \bar{C}^{-1} C_y + HA\,(A^T \bar{C}^{-1} A + P)^{-1} (HA)^T, \qquad (6)$$

$H = C_y^T C^{-1}$, and C_o the variance of y. This expressions (6) inserted in eq. (3), may then be used for gross-error detection.

The method must be used carefully. With gravity data, which generally are supposed not to be affected by biases, the estimate y is calculated without using the value y_{obs} itself. The reason is, that in case σ_y^2 is very small with respect to C_o, then the values of y_{obs} and $\tilde{y}$ will be very close. (This is the basis for the collocation method). This strategy should also be used with altimeter data if many observations are available and all tracks cross each

other.
One should also be aware, that a gross-error may contaminate data in the neighbourhood. The detection and removal of gross errors is therefore in many case a process with several iterations.
The best situation would naturally be if we had a completely independent set of quantities, which could be used to calculate $\tilde{y}$. This is the situation we have, if we have gravity observations in the same area as the altimeter observations. The following two examples will illustrate this.

4. Gravity field modelling and gross-error detection in two areas in the Baltic Sea.

Geoid modelling has been carried out for the Baltic Sea by Andersson and Scherneck (1981) and Vermeer (1983, 1984) using satellite altimetry and by Tscherning (1982, 1983, 1985), Tscherning and Forsberg (1986), Forsberg (1990) using gravity data as the primary source. Gross-errors were eliminated as a preparation for these studies using the methods discussed in section 2, i.e. by inspecting data on individual tracks or by contouring. Most detected errors were found close to small islands.
A renewed interest for gravity field modelling in the Baltic Sea has come partly as a preparation for the ESA ERS-1 mission which will carry an altimeter, and partly due to the wish for having a more precise geoid in the Nordic Area. Here the most recent geoid (Forsberg, 1990) has been calculated by the FFT method exclusively from gravity data. In order to continue this effort, gravity data are needed all over the Baltic Sea. This need is most easily fulfilled by "converting" the altimeter derived geoid heights to gravity anomalies using least squares collocation.
Since the altimeter data had already been used in previous investigations, some gross-errors were already eliminated. But we had never analysed the data as careful as done in those studies dedicated exclusively to the study of the geoid in the Baltic Sea.
In order to test the method discussed in section 3, we therefore selected two typical areas, one with much gravity data and one with few, see Fig. 8 and 16. First the contribution from the IFE88E2 (Basic et al., 1989) spherical harmonic expansion, complete to degree 360 was removed. A covariance function used in the southern part of the Baltic was used,

$$cov(P, Q) = \sum_{i=2}^{\infty} \sigma_i \left(\frac{R^2}{rr'}\right)^{i+1} P_i(\cos\Psi)$$

where P, Q are points with spherical distance Ψ, P_i are the Legendre polynomials, R the mean radius of the Earth, r, r' the radial distances of P, Q respectively, from the earth's center, $R-R_B$ = 9072 km, and

$$\sigma_i^2 = 0 \quad , \quad i = 2 - 9$$

$$\sigma_i = 0.0003 \cdot i \quad , \quad i = 10\text{-}360$$

$$\sigma_i = \frac{A}{(i-1)(i-2)(i+4)} \left(\frac{R_B}{R}\right)^{2i+2} \quad , \; i > 360$$

The implied (residual) geoid and gravity standard deviations were 0.23 m and 7.7 mgal, respectivily.

The altimeter data were first used for gross-error detection with eq. (4) and to predict the gravity data in the area shown in Fig. 8 and 16. Some of the errors also affected the gravity prediction, so that differences between observed and computed gravity values close to an erroneous point reached - 92 mgal. The gross-errors found are shown in Table 1, Fig. 6 and 15, and the largest were eliminated. In a second step the gravity data was used and a few more errors was found. Finally for the northern area, all the available gravity data were used. In Fig. 1-16 are shown geoid and gravity maps, and maps showing the error estimate produced from 4 different collocation runs. Note the changes both in the gravity and geoid maps, and in the error estimates. In Table 2 are shown estimates of the bias and bias errors for tracks which pass through both areas. (There were totally 28 tracks in the southern and 24 tracks in the northern area). A considerable decrease in the error estimate occurs when gravity values are used. Most noteworthy is also the large (25 cm) but systematic change in the individual bias estimates. Note also that the bias estimates in the two areas not are statistically different. However, there is a mean difference of approximally a decimeter, which is significant. This difference has the same magnitude as the difference in sea surface topographic height between the two areas, see Ekmann and Maekinen (1990).

Using altimetry alone, gross-errors of magnitude above k x 0.29 m may be detected. Using additional gravity data this limit is decreased to k x 0.21 m. This shows how the additional use of gravity data may improve our ability to detect gross-errors, and naturally also that the estimate of the geoid is improved considerably compare Fig.(3),(4),(11),(12).

It is also interesting to see the changes in the geoid when gravity data is introduced. Fortunally the changes are pretty well within the bounds given by the error estimates. On the other hand it is encouraging to see that gravity anomalies may be estimated with an error of only 3 mgal from altimeter data alone. This should naturally be seen in relation to the magnitude of the signal standard deviation of only 7.7 mgal for the residual gravity anomalies. These results are in agreement with results found in Knudsen (1987).
When adding gravity values, the error decreases to the gravity noise estimate in areas where the gravity values are located. However only a small improvement (typically from 4 to 3 mgal), see Fig (6),(8),(15),(16), are found in areas where no gravity is available.

5. Conclusion

Altimeter data in the two coastal regions studied contain between 0.5 - 1.0% gross errors. These errors must be eliminated before the data is used for gravity modelling. The formation of histograms, contouring of data and local least-squares collocation may be used for gross-error detection. However, local biases or tilts should be removed first.
The use of additional, independent information, like gravity data may significantly improve the possibility for the detection of errors. Also, the contribution from the best possible reference potential (as well as local topographic effects) should be subtracted in order to obtain a signal variance (C_o) as small as possible.
The statistical foundation of the use of least-squares collocation for gross-error detection needs improvement despite recent progress (Krakiwsky and Biacs, 1990). Especially the introduction of parameters complicates the picture.
It is the use of LSC which permit the combination of altimeter and gravity data. The results from the Baltic Sea demonstrates the improvements in the determination of the geoid, which may be achieved by combining altimeter and gravity data. The decrease of the error of estimated gravity values in areas without gravity data is small, but important.

Table 1. Detected Gross-errors, k = 3.

Northern block (468 obs.)		From altimetry only.	From altimetry and gravity
Obs. no.	Error (m)		
5740104	0.85	no	yes
4260143	-0.90	yes	yes
7130155	0.73	no	yes
7750125	2.18	yes	yes
7750143	-0.97	yes	yes

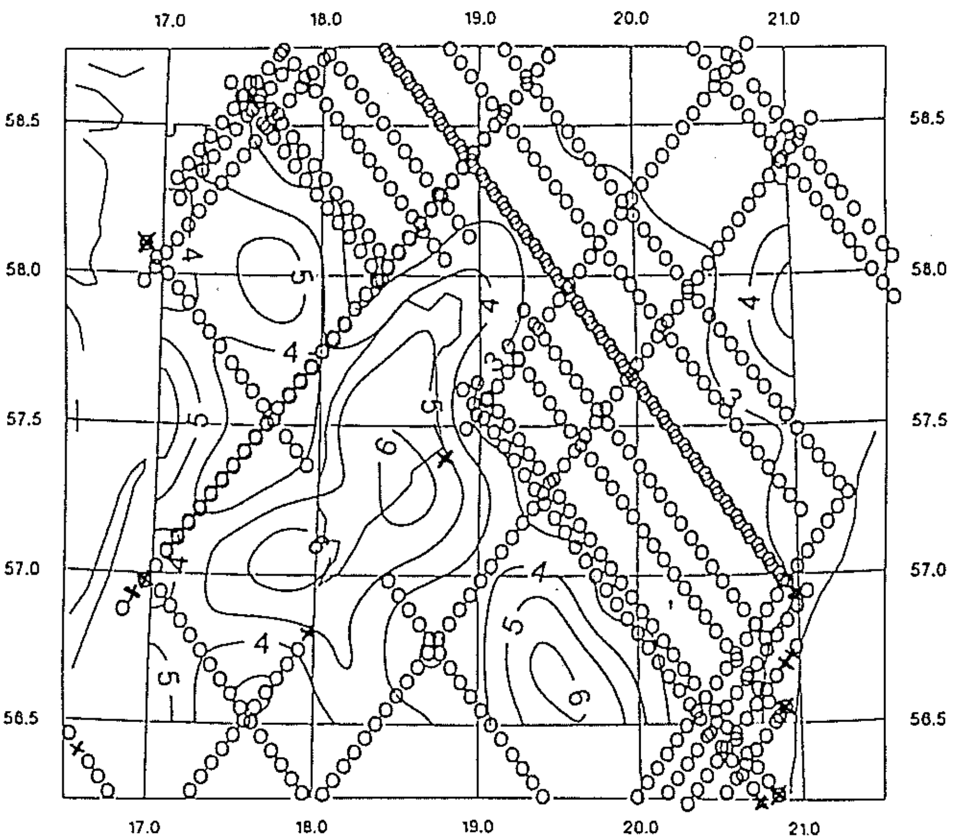
Southern block (764 obs.)		From altimetry only.	From altimetry and gravity
Obs. no.	Error (m)		
5550424	0.99	yes	yes
7560191	1.02	no	yes
7560192	-0.96	no	yes
7610084	1.31	yes	yes
8900083	1.09	yes	yes
11910001	-2.34	yes	yes
12770001	-2.08	yes	yes

Tabel 2. Bias estimates (cm) for tracks in both areas, note drastic decrease in the error estimate of the bias, when gravity is used.

	NORTH AREA						SOUTH AREA					
Observ:	Alt. only		+114 Δg		+593 Δg		Alt. only		+238 Δg		Diff. between best bias estimates	
Track	bias	$\hat{\sigma}$	bias	$\hat{\sigma}$	bias	$\hat{\sigma}$	bias	$\hat{\sigma}$	bias	$\hat{\sigma}$		$\hat{\sigma}$
191**	-60	±22**	-75	±20	-75	±13	-70	±14	-94	±10	19	±16
215	-73	17	-91	14	-96	8	-86	14	-110	10	14	13
234	-63	15	-89	10	-95	9	-78	14	-103	10	8	13
258	-70	15	-97	10	-103	8	-95	14	-121	10	18	13
277	-62	15	-87	9	-92	8	-75	19	-93	17	1	19
459	-38	17	-55	10	-59	8	-39	14	-65	10	6	13
521	-63	15	-89	9	-93	8	-81	18	-99	16	6	18
545	-67	15	-92	10	-95	8	-90	14	-113	10	18	13
703*	-110	18*	-25	10	-31	8	-110	14	-38	10	7	13
722	-57	15	-84	10	-89	9	-78	14	-103	10	14	13
746	-62	15	-89	10	-95	9	-77	14	-103	10	8	13
Geoid estimate minimum error:		15		9		8		14		10		

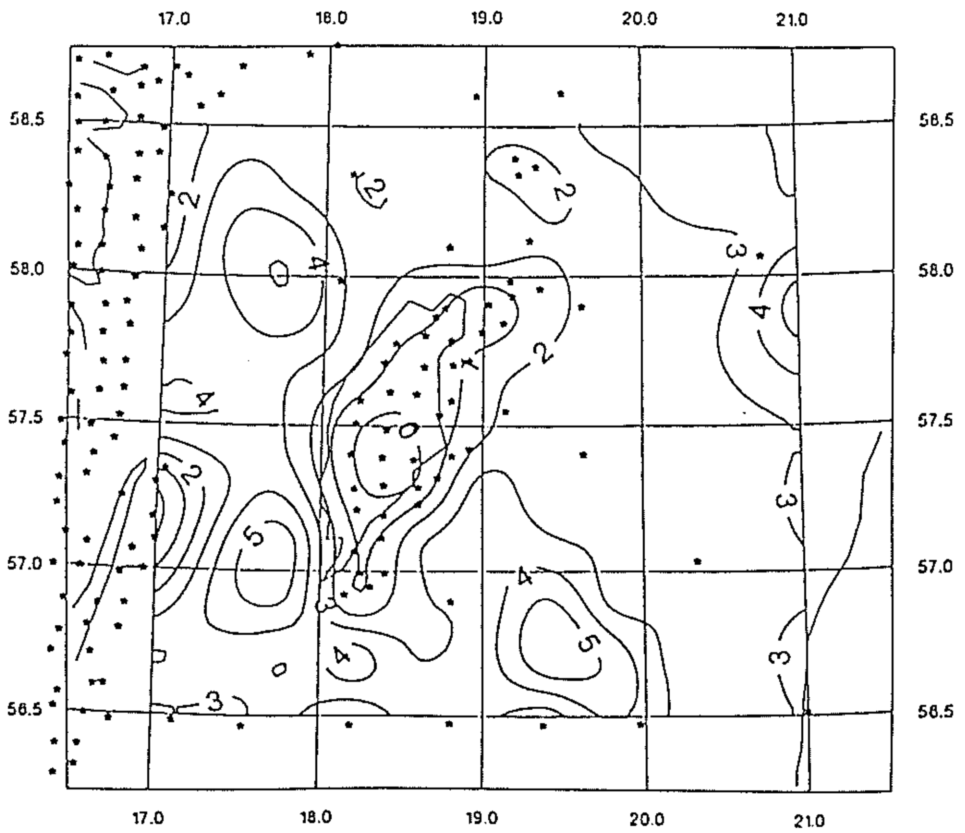
* LOOSE TRACK ** 1 OBS. ONLY!

Fig. 15



ERROR OF GRAVITY FROM ALTIMETRY, C.I. 1.0 MGAL X=GROSS-ERROR

Fig. 16



ERROR OF GRAVITY FROM ALTIMETRY AND GRAVITY, C.I. 1.0 MGAL AND GRAVITY.