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THE USE OF HEIGHT DIFFERENCES DETERMINED BY THE GPS IN THE CONSTRUCTION PROCESS OF THE FIXED LINK ACROSS THE GREAT BELT

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SUMMARY

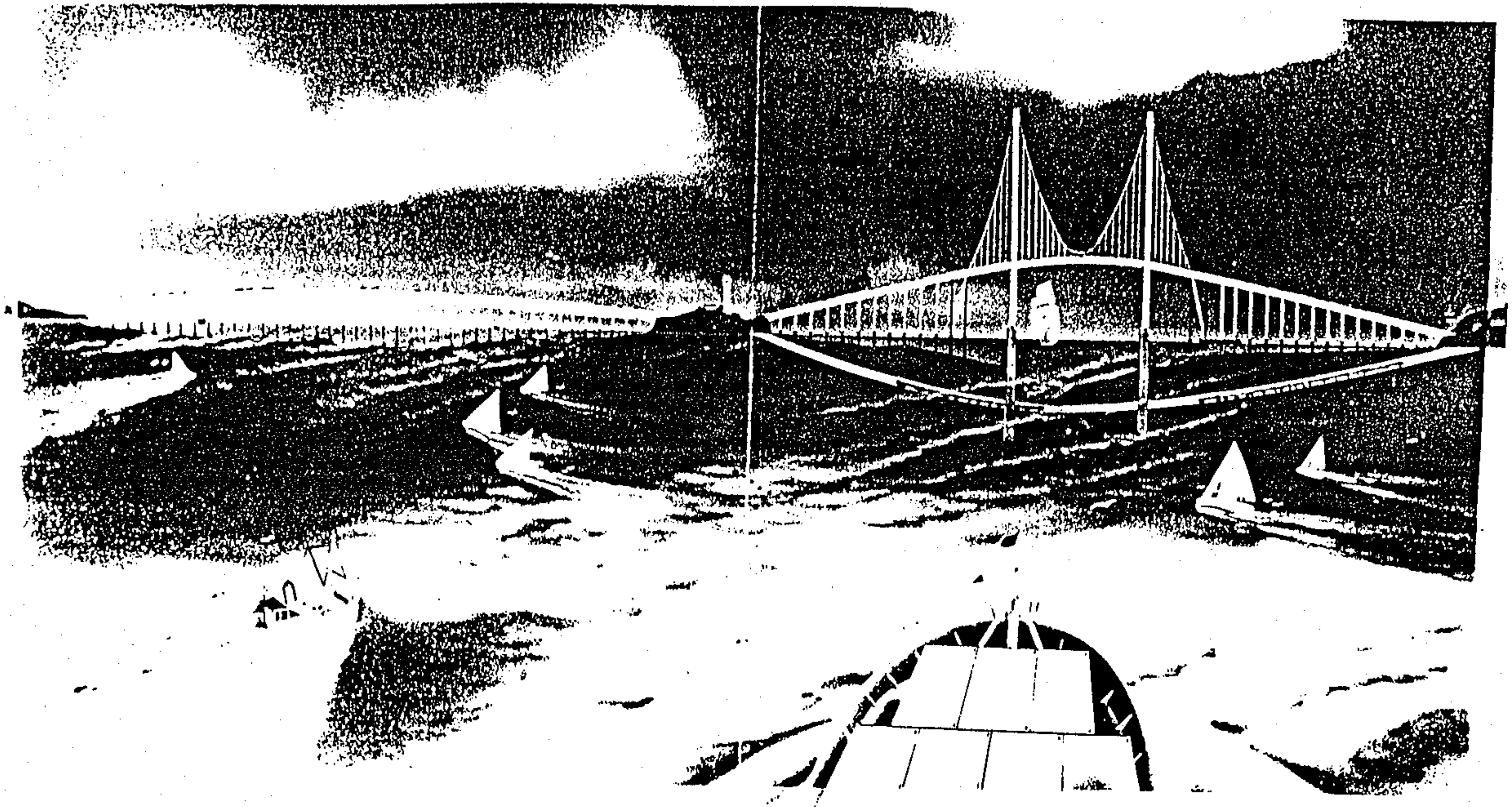
GPS determined heights from which geoid heights are subtracted have been used for the determination of height differences with an accuracy of 1-2 ppm across the Great Belt where a fixed link is under construction. A comparison of the height differences with results obtained more than 50 years ago using hydrostatic levelling show an agreement of 4-5 cm over the 20 km distance. Despite the good agreement, this is not sufficiently accurate to determine if land movements have taken place in the period.

1. INTRODUCTION

The Fixed Link across the Great Belt is in the process of being build, and it is supposed to be completed in 1996, see Fig. 1. Denmark is an area affected by the postglacial rebound (Bedsted Andersen et al. 1974). In general a tilting of Denmark is taking place. The axis of zero tilt goes approximately from Northwest to Southeast, with a subsidence of 1-2 mm per year in the South-West of Denmark. These movements have been determined by precise levellings in 1885-94, 1943-53 and the third levelling takes place at present.

The height difference across the Great Belt was determined in 1938 by hydrostatic levelling (Nørlund, 1945), and has not been repeated. It will be repeated this spring as a part of a cooperative project with the University of Hannover, sponsored by EEC.

The authority responsible for the Fixed Link (Storebæltsforbindelsen A/S) requested an investigation of possible land movements across the Belt since 1938. Due to our positive results in determining orthometric height differences using space positioning methods and geoid information in Greenland (Forsberg,Madsen, 1981), we decided to use the same method to establish a new height connection across the Belt. Here we are in the advantageous situation that GPS has become operational, providing 3-dimensional position differences at the level of 1 ppm for short distances like these from the coast to the island of Sprogø and across the Belt, see Fig. 1 and 2.



Storebælt

Fig. 1. The Fixed Link across the Great Belt

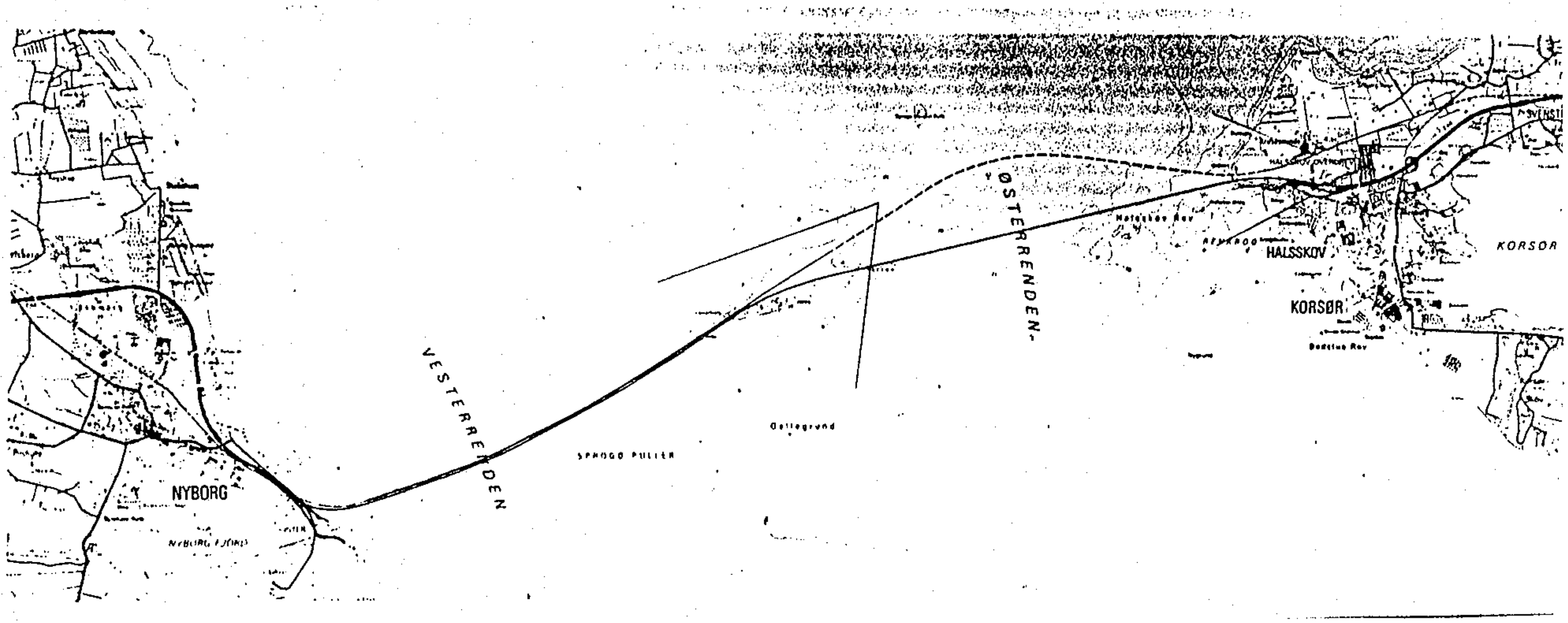


Fig. 2. The Fixed Link across the Great Belt

A precise geoid determination is also possible, not so much due to the distribution of gravity data, see Fig. 3, needed for the computation, but more due to the extreme smoothness of the gravity field in Denmark, and across the Belt, see Fig. 4.

In section 2 we will briefly outline the acquisition and processing of the GPS data, and the method used for calculation on the geoid. The area has existing geodetic control, which will be described and compared with the GPS derived results in section 3. Finally in section 4 we will present the evaluation of the results, and discuss the quality, and future investigations.

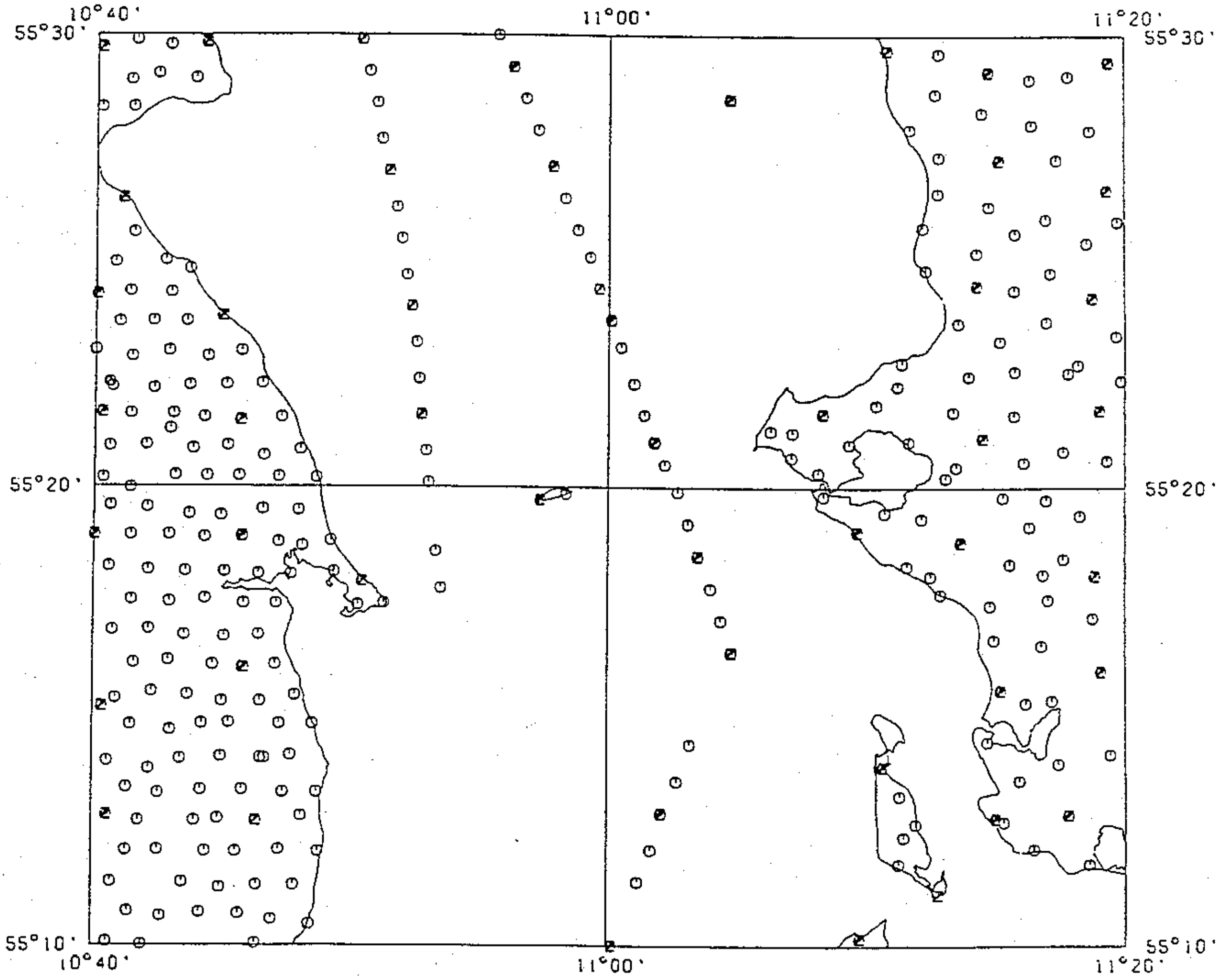


Fig. 3

Gravity coverage in the Great Belt area
○ Gravity station
● Gravity observations used in geoid calculation

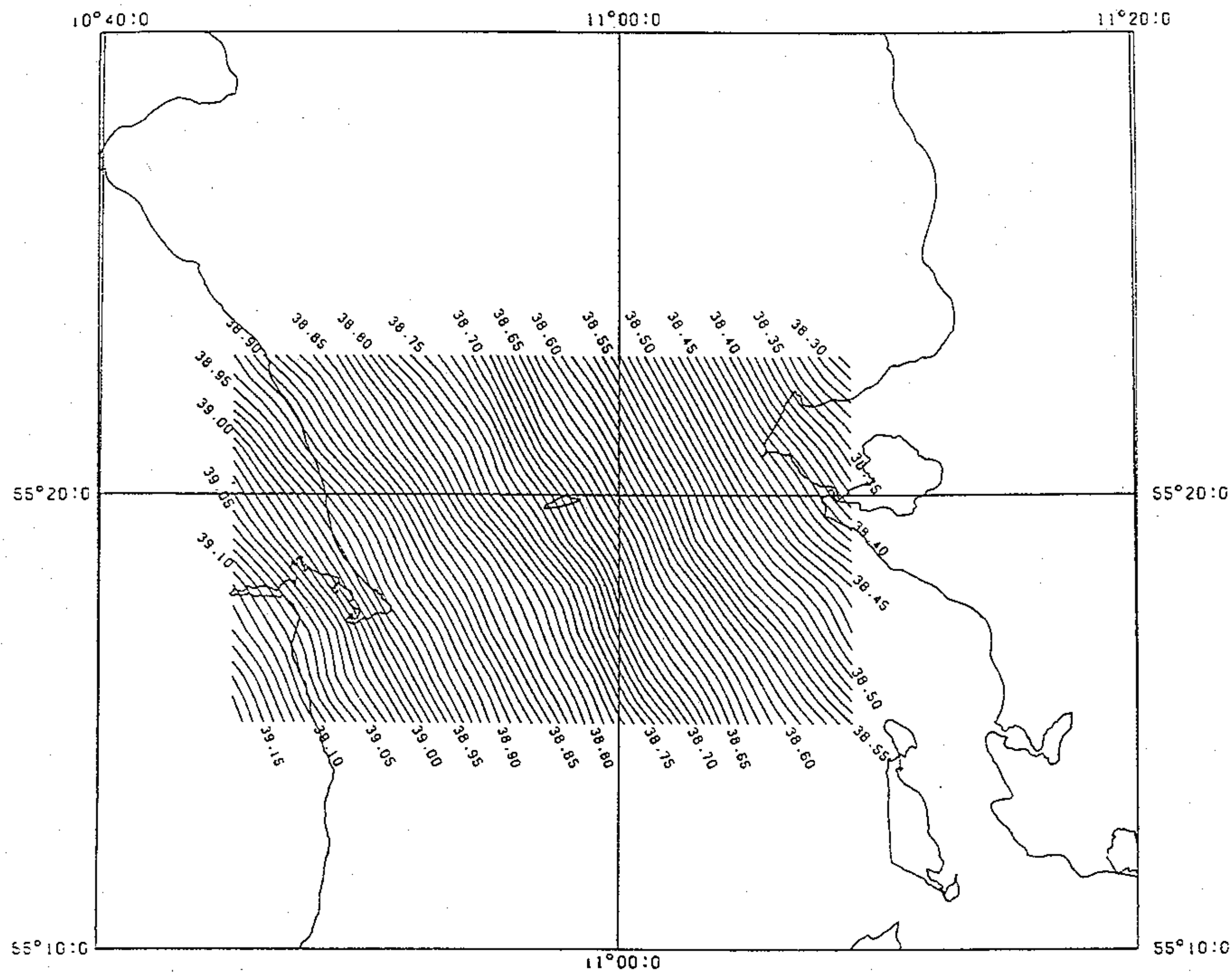


Fig. 4. Geoid computed by the LSC method

2. METHOD FOR THE DETERMINATION OF ORTHOMETRIC HEIGHT DIFFERENCES

Space methods determine 3-dimensional position differences, which may be used to compute ellipsoidal height differences, $h_1 - h_2$, where the subscript indicate that we consider two different observation sites.

A gravity field modelling method (Tscherning, 1981) provides us with geoid height (or height anomaly) differences, $N_1 - N_2$. From these quantities, we get the orthometric height difference

$$H_1 - H_2 = h_1 - h_2 - (N_1 - N_2) \qquad (1)$$

(A small modification is necessary if normal heights are used instead of orthometric heights, but this is without consequence in our case due to the heights of the stations involved).

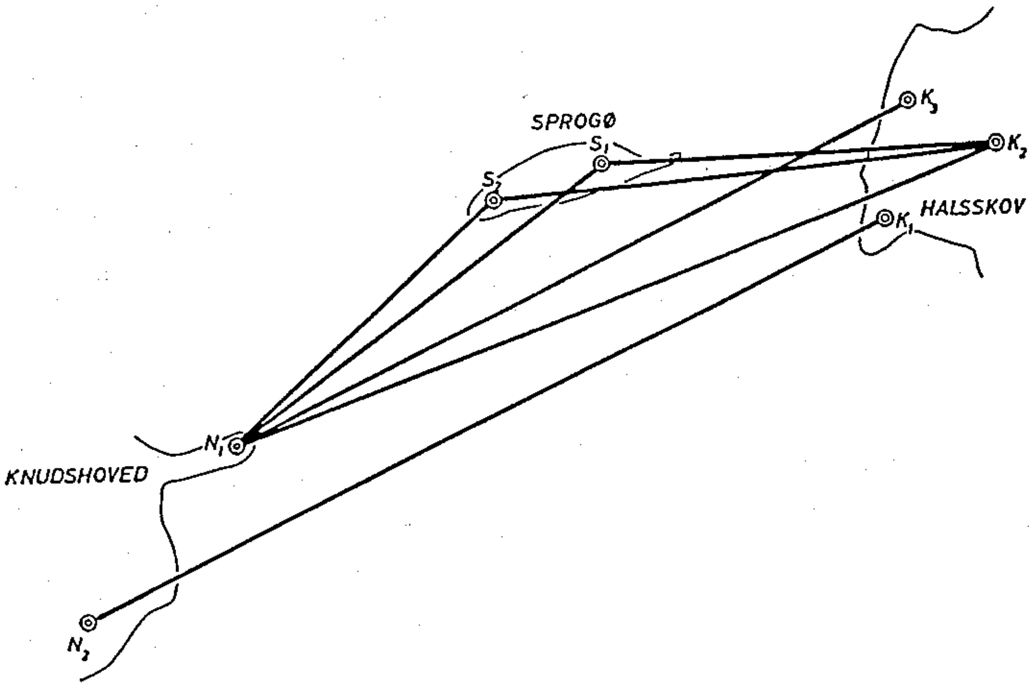


Fig. 5. Observed GPS vectors across the Great Belt

GPS-observations were executed in the period 10 Oct. - 14. Oct., 1988. 7 stations were occupied, 3 on Sjælland, 2 on Sprogø and 3 on Fyn, see Fig. 5. Two TI4100 GPS receivers were available for the project. The stations were in general occupied in 3-5 hours, but the position differences were calculated using observation periods between 40 and 190 minutes. The program GEOMARK provided by Texas Instruments were used to process the data.

The geoid computation was made in two different ways, the method of Least-Squares Collocation (LSC) and the Fast Fourier Transform (FFT) method (Tscherning and Forsberg, 1986). The LSC method is an optimal estimation method, which requires a large numerical effort, and it is generally only possible to use 2-3000 data points in the calculation. Consequently only data in the distance of 100-200 km from the area of interest are used. The FFT method may take large amounts of data into account, but the data must be gridded.

In principle the computation of the geoid requires that a global set of gravity data is available. However, by first subtracting a spherical harmonic expansion, and later adding the effect of this to the result, a limited area may be used, (Tscherning, 1983). As spherical harmonic reference fields were used the IFE88E2 field (Denker, 1989) and the OSU89B field (R.H. Rapp, personal communication 1989). Both fields are complete to degree 360. Topographical effects were also taken into account, but had an influence below 1 cm on the results.

A part of the gravity data is shown in Fig. 3. Note that only a subset of the available values were used. While the FFT method does not permit the computation of an error estimate, this is possible for the LSC method (Moritz, 1980). The error estimate for the geoid height difference across the Belt were 1-2 cm. This should be seen in view of the smooth geoid, which only varies ± 0.33 m when the contribution from the spherical harmonic expansion has been subtracted. The IFE88E2 field was used with the LSC computation, while the FFT computation used the recently available OSU89B field (Forsberg, 1990).

The geoid computed by the LSC method is shown in Fig. 4.

3. COMPARISON OF GPS DERIVED RESULTS WITH EXISTING GEODETIC CONTROL

Hydrostatic levelling was executed in 1938, (Nørlund, 1945), between points which were connected by precise levelling to the stations occupied by the GPS receivers. The height difference determined by this method is supposed to have an error less than 0.5 cm.

In the same points horizontal control was available, and the position differences agreed within 1 ppm.

Table 1 summarizes the results obtain using eq. (1), which combines the GPS derived ellipsoidal heights and the geoid undulation differences. The results using the LSC and the FFT method are both found. Note that a systematic difference is found increasing from east to west (see Fig. 5 for station location).

Table 1. Comparison of elevations determined by precise levelling including hydrostatic levelling with elevations determined by GPS supplied with geoid heights using two different methods and reference fields.

Reference station	New station	Old heights-GPS heights (cm) Geoid: LSC	Old heights-GPS heights (cm) Geoid: FFT	Distance (km)
K1	N2	5.6	3.6	18
K2	N1	7.9	5.1	17
K2	S1	2.6	2.4	9
K2	S2	2.0	2.2	10
K3	N1	7.4	5.1	16
S1	N1	4.4	1.9	8
S2	N1	6.7	3.7	7

4. EVALUATION OF THE RESULTS AND FUTURE INVESTIGATIONS

A statistical test of the results in Table 1, under the hypothesis that a land movement has taken place must be rejected. It is possible that improved GPS observation technique, (using more receivers and longer observation period) and a more precise geoid (with reduced long wavelength errors, and better distribution of the gravity data at sea, see Fig. 3), would have made it possible to verify a movement or a stable situation, respectively. The strongest test would naturally be a new hydrostatic levelling, which fortunately is scheduled to take place this spring, 1990.

5. REFERENCES

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