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METHODS FOR THE REGIONAL GRAVITY FIELD MODELLING FROM SST AND SGG DATA

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Abstract: The use of linear approximation methods, integral formula techniques and frequency domain methods for gravity field modelling using satellite-to-satellite tracking or satellite gravity gradiometry data have been reviewed. The linear approximation methods have the advantage that they can be used directly with the original data, and that they generally permit the simultaneous estimation of parameters like instrument biases and the use of data at the Earth's surface. A method like least squares collocation also permits the computation of error estimates.

The approximation methods generally require a large numerical effort, which may be reduced by considering small areas or computing mean values of observations. From local solutions based on such limited datasets, gridded observational values may be computed, which then may be used as input to regional or global frequency domain modelling procedures.

For all methods it is of advantage first to subtract from the observations the contribution from the best available spherical harmonic expansion and the (residual) topography. This will smooth and decorrelate the data, and thereby reduce the error of estimated quantities and ease the detection of gross errors in the data.

1. Introduction

Many methods have been proposed for the regional representation of the Earth's gravity potential. They have primarily been designed as to take advantage of data collected at the surface of the Earth. However, several of the methods have such a general character that they also may be used with data observed above the Earth's surface such as satellite-to-satellite tracking (SST) data or satellite (or aircraft) gravity gradiometer (SGG) data.

In this report we will review methods which have already been proposed for the regional representation (or "modelling") of the gravity potential using ground and/or satellite data. As usual we will "model" the so-called anomalous gravity potential,

$$T = V - U \qquad (1.1)$$

where V is the gravity potential and U a "normal" or reference potential. We will suppose that U includes the attraction of the atmosphere, the Moon, Sun, the planets etc. Then T is a harmonic function in space.

For the observations we will suppose that they are associated with T through linear functionals, L_i, $i=1,\ldots,n$, and contingently depend linearly on a number of parameters, X_j, $j=1,\ldots,k$. We then have for an observation y_i

$$y_i = L_i(T) + \sum_{j=1}^{k} A_{ij} X_j + e_i, \quad i=1,\ldots,n \qquad (1.2)$$

where A_{ij} is a vector of known constants and e_i is the observation error. In section 2 we will derive mathematical expressions for the functionals L_i, which relate SST and SGG data to T.

The anomalous potential may be regionally modelled by three different types of methods:

(a) linear approximation methods,
(b) integral formula methods, and
(c) frequency domain or spectral methods.

The approximation methods differ with respect to which and how many base functions are used (compared to the number of observations used), and which minimum principle (if any) is used to obtain the approximation. These various possibilities, their advantages and disadvantages, will be discussed in section 3.

For data at the Earth's surface integral formula methods are very efficient, and give good results if well distributed data are available, as will be the case for a longer lasting SST or SGG mission. However, as will be discussed briefly in section 4, these methods seem not to be directly usable for data at satellite altitude due to problems with the downward continuation, which amplify the high frequencies.

In the frequency domain the high (and low) frequencies may be cut off, and methods developed using FFT for airborne gradiometry may be used. This is discussed in section 5.

For all methods it is of advantage to work with data, which are as smooth and uncorrelated as possible. Smoothing and decorrelation may be achieved by subtracting from the data the contribution from the best possible spherical harmonic expansion and from the (residual) topography. Hereby liniarization errors and errors due to the use of "flat earth approximation" are minimized. Gross errors are more easily detected in the data, and parameters are better determined. This is discussed in section 6.

Approximation and integral formula methods for regional or local modelling have been reviewed e.g. in Tscherning (1981, 1986), and spectral methods in Schwarz et al. (1989). The regional modelling problem has been the object of several special study groups (SSG) within the International Association of Geodesy (IAG), see Schwarz (1984), Dufour (1984) and Tscherning (1988). Numerical tests and comparisons using ground data are reported in Schwarz (1983), Kearsley et al. (1985), using several of the methods which also may be used with SGG and SST data. The use of these data has also been studied by IAG (Rummel, 1988) both globally and regionally, and is currently being studied, see e.g. the working programs of the relevant SSG's in the 1988 issue of the Geodesist's Handbook (Tscherning, 1988a).

Due to the extensive research activity currently going on within the subject area of this paper, we have not been able to review all the relevant papers in detail. However, the reader will find an extensive bibliography as an appendix to this paper covering primarily the period since 1983. References to earlier literature may be found in the issues of "Travaux de l'Association Internationale de Géodésie", published after each IAG general assembly.

2. Observation functionals

In this section we will derive expressions for the linear functionals relating SST and SGG observations to the anomalous potential. A more detailed derivation, considering several cases of optimal orbit configurations is found in Vermeer (1989b).

2.1 Satellite-to-satellite tracking

We have two satellites (1) and (2) located at $\underline{x}_1$ and $\underline{x}_2$, moving with velocities $\underline{v}_1$ and $\underline{v}_2$ and accelerations $\underline{a}_1$ and $\underline{a}_2$. As a convention for the relative position, velocity and acceleration we use

$$\underline{x}(t) = \underline{x}_{12}(t) = \underline{x}_2(t)-\underline{x}_1(t) \qquad (2.1)$$

$$\underline{v}(t) = \underline{v}_{12}(t) = \underline{v}_2(t)-\underline{v}_1(t) \qquad (2.2)$$

$$\underline{a}(t) = \underline{a}_{12}(t) = \underline{a}_2(t)-\underline{a}_1(t) \qquad (2.3)$$

The range between the satellites will be denoted $x(t)$, with time derivatives $v(t)$ and $a(t)$. They are not equal to the modulus of $\underline{v}(t)$ and $\underline{a}(t)$.

The quantity which is being measured is the range between the satellites, plus a constant, unknown bias term. (The measurement may actually be the time average over a certain period, but we will here consider the more likely case of measurements related to a specific time, t). These measurements are produced at a constant rate of e.g. 4 seconds, and we will consider the measurements as being uncorrelated.

Differencing these primary measurements produces range rates, which however are correlated. The bias term is eliminated. A second differencing may be performed, producing range rate rates, closely related to line-of-sight accelerations. Also these quantities are correlated, see Vermeer (1989).

It is easy to derive that the second time derivative of the range, i.e. a(t), consists of two contributions: the line-of-sight component of the (vectorial) acceleration difference and a "centrifugal" effect of the transversal relative velocity,

$$a = \left[\langle \underline{a} \cdot \underline{x} \rangle + \left[\frac{|\langle \underline{v} \wedge \underline{x}\rangle|}{|\underline{x}|}\right]^2 \right] / |\underline{x}| . \qquad (2.4)$$

If we disregard non-inertial forces, then the first part of the equation is most important, since

$$\underline{a} = \mathrm{grad}\, V(\underline{x}_2) - \mathrm{grad}\, V(\underline{x}_1), \qquad (2.5)$$

and its directional component is the linear functional needed in eq. (1.2). But we also have a dependence on the orbit.

If we linearize the satellite orbit, instantaneously described by a state vector $(\underline{x}(t), \underline{v}(t))^T$, we obtain for small differences:

$$\begin{bmatrix} \delta\underline{x}_s(t) \\ \delta\underline{v}_s(t) \end{bmatrix} = (S^s)^t_{t_0} \begin{bmatrix} \delta\underline{x}_s(t_0) \\ \delta\underline{v}_s(t_0) \end{bmatrix}, \qquad (2.6)$$

using

$$\delta\underline{x}_s = \underline{x}_s - \underline{x}_s^0 \quad \text{and} \quad \delta\underline{v}_s = \underline{v}_s - \underline{v}_s^0,$$

s=1,2 identifying the satellite considered. S is the state transition matrix, obtained numerically or analytically.

This defines our first subvector of unknown parameters, the initial state vector $(\delta\underline{x}(t_0), \delta\underline{v}(t_0))^T$.

We have from eqs. (2.5) and (1.1):

$$\underline{a} = \mathrm{grad}\, U(\underline{x}_2) - \mathrm{grad}\, U(\underline{x}_1) + \mathrm{grad}\, T(\underline{x}_2) - \mathrm{grad}\, T(\underline{x}_1) \qquad (2.7)$$

Here we must include in the calculation of grad U, which may change considerably from point to point, compared to grad T, the dependence on the two locations, $\underline{x}_1$, and $\underline{x}_2$. Hence,

$$\mathrm{grad}\, U(\underline{x}_s) = \mathrm{grad}\, U(\underline{x}_s^0) + \mathrm{grad\,grad}\, U(\underline{x}_s^0)\cdot\delta\underline{x}_s \qquad s=1,2, \qquad (2.8)$$

so that we have

$$\underline{a} = \mathrm{grad}\, U(\underline{x}_2^0) - \mathrm{grad}\, U(\underline{x}_1^0) + \mathrm{grad\,grad}\, U(\underline{x}_2^0)\cdot(S_x^2)^t_{t_0}\begin{bmatrix} \delta\underline{x}_2(t_0) \\ \delta\underline{v}_2(t_0) \end{bmatrix} - \mathrm{grad\,grad}\, U(\underline{x}_1^0)\cdot(S_x^1)^t_{t_0}\begin{bmatrix} \delta\underline{x}_1(t_0) \\ \delta\underline{v}_1(t_0) \end{bmatrix} + \mathrm{grad}\, T(\underline{x}_2) - \mathrm{grad}\, T(\underline{x}_1). \qquad (2.9)$$

In order to obtain the linear functional associated with the range rate rates, a, (eq. (2.4)), we must compute the scalar product with $\underline{x}/|\underline{x}|$, and the "centrifugal" effect, or dynamic constant term,

$$b_d = |\langle \underline{v} \wedge \underline{x}\rangle|^2 / |\underline{x}|^3 \qquad (2.10)$$

which also depends on the initial state vector.

It naturally gives a considerable simplification, if we may consider the orbit as known. In this case, combining eq. (2.4) and (2.9) we get

$$y_i = a - b_n - b_d =$$
$$= \langle (\text{grad } T(\underline{x}_2) - \text{grad } T(\underline{x}_1)) \cdot \underline{x} \rangle \;/\; |\underline{x}| \qquad (2.11)$$

with

$$b_n = \langle (\text{grad } U(\underline{x}_2) - \text{grad } U(\underline{x}_1)) \cdot \underline{x} \rangle \;/\; |\underline{x}| \qquad (2.12)$$

the reference field term.

One further simplification is possible, namely in the so-called High-Low SST case, a situation which e.g. was studied in Reigber et al. (1987). Suppose the second satellite is flying at a high altitude (7000 km). For this satellite we may consider grad $T(\underline{x}_2)$ to be known, and we have

$$y = a - b_n - b_d - b_h = \langle \text{grad } T(\underline{x}_1) \cdot \underline{x} \rangle / |\underline{x}| \qquad (2.13)$$

where

$$b_h = \langle \text{grad } T(\underline{x}_2) \cdot \underline{x} \rangle / |\underline{x}|. \qquad (2.14)$$

Here the observable y has become simply the projection of the anomalous acceleration vector at the location of the low satellite onto the <u>line of sight</u> $\underline{e} = -\underline{x}/|\underline{x}|$ between the satellites.

If one wants to work with the <u>range-rates</u>, the observation functionals become equal to the ones given above, numerically integrated e.g. from an initial state. The practical implementation of such equations will probably not be very difficult.

2.2 Satellite gravity gradiometry

Satellite gravity gradiometer data are measurements of the second order derivative of V, sampled over a certain time period. However, we will regard the observations as being point related, and collected in a satellite orbit-bound frame, (u,v,w), where the u-axis is directed along the velocity vector, the w-axis points radially outward (to be defined later) and the v-axis is the second axis in a left-handed orthonormal frame.

Applying the gradient operator twice yields

$$\text{grad grad } V = \text{grad grad } U + \text{grad grad } T,$$

which we arrange in a (symmetric) 3 x 3 matrix, as usual.

If we have a pair of measurement directions, $\underline{d}$, $\underline{e}$, we obtain by pre- and post multiplication,

$$y = \langle \underline{d} \cdot \text{grad grad } V \cdot \underline{e} \rangle - \langle \underline{d} \cdot \text{grad grad } U \cdot \underline{e} \rangle$$
$$= \langle \underline{d} \cdot \text{grad grad } T \cdot \underline{e} \rangle \qquad (2.15)$$

or for brevity

$$y_{de} = V_{de} - U_{de} = T_{de}.$$

The calculation of U_{de} depends strongly on the precise orbit, so by writing down eq. (2.15) we use the "known orbit" simplification as in section 2.1. For the derivatives of T, the position dependence is a second order effect.

Generally, when dealing with the gravity field, quantities are conveniently expressed in a local cartesian coordinate system (x,y,z) with x-axis, pointing south, y, east, and z-axis, radially away from the geocenter. The relationship between this system, and the (u,v,w) system may be expressed by a rotation matrix dependent on three angles:

- A: azimuth angle, in the plane orthogonal to the z-axis,
- β: dip angle, in the plane orthogonal to the y-axis,
- γ: roll angle, in the plane orthogonal to the x-axis.

Taking into account that the angles β and γ are small, the full rotation matrix may be written

$$R_{\gamma\beta A} = \begin{pmatrix} -\cos A & -\sin A & -\beta \\ -\sin A & +\cos A & -\gamma \\ -\beta\cos A-\gamma\sin A & \gamma\cos A-\beta\sin A & 1 \end{pmatrix} \qquad (2.16)$$

The rows of this matrix are the vectors $\underline{d}$ and $\underline{e}$ needed, when expressing the observation functionals (2.15) in the usual (x,y,z) system.

The second-order derivatives in the (x,y,z) system may then be expressed as a function of (geocentric) latitude, (φ), longitude (λ) and radius vector (r), see Tscherning (1976).

3. <u>Approximation methods for regional gravity field modelling</u>

3.1 General considerations

The <u>linear</u> approximation methods determine a representation $\tilde{T}$ of T as a linear combination of given base functions f_i,

$$\tilde{T}(P) = \sum_{i=1}^{m} a_i f_i(P) \qquad (3.1)$$

where P is a point in space, and a_i are unknown constants. Using T(P) we may find an estimate of the value of a functional L(T), by

$$\widetilde{L(T)} = L(\tilde{T}) = \sum_{i=1}^{m} a_i\, L\,(f_i(P)) = \sum_{i=1}^{m} a_i f_i(L) \quad (3.2)$$

where we have introduced the notation Lf(P) = f(L) in order to shorten the already long expressions. An estimate of the geoid height in a point P is then simply calculated using Bruns' formula,

$$\tilde{\zeta}_P(T) = \sum_{i=1}^{m} a_i f_i(P)/\gamma,$$

where γ is the normal or reference gravity at P. The gravity disturbance is computed by calculating the radial derivative of $\tilde{T}$.

Since T itself is a harmonic function (and regular at infinity) the functions f_i are generally also harmonic. However, non-harmonic functions have been proposed for global applications (Meissl, 1981), (Baker, 1988), and for regional applications (Hardy and Nelson, 1986). The regional use of the non-harmonic functions proposed by Hardy and Nelson (1986) has been investigated in Priovolos (1988) with a negative result. We will in the following only consider harmonic base functions, but many of the results given subsequently are also valid for non-harmonic base functions.

The harmonic base functions may generally be characterized as potentials of certain types of "masses": point masses, dipoles, multipoles, surface layers, or as derivatives of such functions. The constants a_i may then be given a (heuristic) physical interpretation as values of certain masses. We have what is called a "source" representation of T. (Note, that other types of representations may also be used to calculate estimates of mass density values using eq. (3.2), see e.g. Tscherning and Strykowski (1988).)

Also, the functions may be derived from the integral kernels, which solve a boundary value problem for T. For gravity anomalies, Δg, the Stokes function, $S(\psi,r)$, solves the boundary value problem, (Heiskanen and Moritz, 1967, Chp. 2),

$$T(P) = \iint_{\sigma} S(\psi,r)\Delta g(Q)d\sigma_Q \qquad (3.3)$$

where ψ is the spherical distance between P and the point Q, and r is the distance of P from the origin. Rewriting the integral as a sum over a grid of points Q_i (or equiangular blocks with midpoints Q_i and area Δ_i), then we obtain an expression like eq. (3.1)

$$\tilde{T}(P) = \sum_{i=1}^{m} \widetilde{\Delta g(Q_i)}\ S(\psi_i,r)\cdot\Delta_i \ . \qquad (3.4)$$

Here we have what is called an "effect" representation of T. (ψ_i is the spherical distance between Q_i and P).

The selection of the base functions may also be based on considerations concerning the smoothness of T. This is the basis for the well known methods of spline interpolation, where one minimalizes the integral of the square of the second order derivatives (curvature) of the function. However, in theory the "best" approximation is viewed as the one which gives the smallest error, or for which the norm of the difference $\tilde{T}$-T is minimum. But <u>which</u> norm should be minimal, is not provided by the theory.

In practice, a comparison of methods is always made in terms of the mean square difference between observed and computed quantities. This makes therefore the quantity to be minimized dependent on the quantity we want to estimate, i.e. T, which is only incompletely known.

3.2 The collocation methods

A method which minimalizes the mean square error is sometimes denoted a method of optimal (linear) estimation. We will here very briefly describe the features of such a method, known in geodesy as least-squares collocation (l.s.c.). This method will then be used as a basis for a comparison with other methods, which use other principles for the selection of the base functions and the determination of the constants a_i in eq. (3.1).

In the most general setting (which includes spline function - like principles of approximation) we regard T as being an element of an infinite dimensional space of harmonic functions. We will also suppose that the space is equipped with an inner product $(\cdot,\cdot)$ and corresponding norm, and has a reproducing kernel. This kernel is a function K(P,Q) of two variables, so that for any element (f) of the space it has the reproducing property: $f(Q) = (K(P,Q), f(P))$.

Suppose that our observations $y_i = L_i(T)$ have no error and do not depend on any parameters. Then $\tilde{T}$ may be determined, so that it has minimum norm (maximal smoothness) and agrees exactly with the observations, $L_i(\tilde{T}) = L_i(T)$. If T is an element of the space, then also $||T-\tilde{T}||$ will be minimal.

The mathematical theory (see e.g. Moritz (1980)) leads to the selection of the so-called Riesz-representers of the observation functionals as base functions.

$$f_i(P) = L_i(K(P,Q)) = K(P,L_i). \qquad (3.5)$$

(For certain norms, this gives base functions identical to these selected based on the wish to construct a certain "source" or "effect" representation, see Tscherning (1983).).

The number of base functions will obviously be equal to the number of observations, m. So the coefficients a_i are determined by solving a system of equations with m unknowns,

$$y_i = L_i(T) = L_i(\tilde{T}) =$$

$$= L_i[\sum_{j=1}^{m} a_j K(P,L_j)] = \sum_{j=1}^{m} a_j K(L_i L_j) \qquad (3.6)$$

This is the method of minimum norm collocation. If T is an element of the space, the method even permits us to compute error bounds,

$$|L(\tilde{T})-L(T)| \le ||T|| \cdot \left[K(L,L)-\{K(L,L_i)\}^T\{K(L_i,L_j)\}^{-1}\{K(L,L_j)\}\right] \qquad (3.7)$$

The observation errors may also be taken into account by minimalizing simultaneously the noise variance. As a consequence will the noise variance-covariance matrix be added to the matrix $K(L_i,L_j)$ in eq. (3.6).

In the same manner the parameters X_i in eq. (1.2) may be determined by using as an extra condition, that a norm of the X_j-vector is minimized, see Moritz (1980, eq. (30-27)). The system of equations (3.6) is then extended with k extra rows and columns, i.e. we have to solve a k+m dimensional system of equations.

Reproducing kernels corresponding to various explicitly given norms are given in Tscherning (1972). Another form of a norm is given implicitly (as a function of T itself) in case we require the integral of $(\tilde{T}-T)^2$ to be minimalized in a statistical sense, see Sanso' (1986). In this case the reproducing kernel is the so-called empirical covariance function associated with T.

If we require the integral of $(\tilde{T}-T)^2$ to be minimalized for all configurations of observation points which may be obtained by rotating the configuration of observation points around the Earths center (see Heiskanen and Moritz, 1967,Ch. 7), then this empirical covariance function becomes rotational invariant, and reasonably easy to determine. Let the spherical harmonic expansion of T be (in spherical approximation)

$$T(\phi,\lambda,r) = \frac{GM}{r}\sum_{i=2}^{\infty}\left(\frac{R}{r}\right)^i \sum_{j=-i}^{i} \bar{C}_{ij} Y_{ij}(\phi,\lambda) \qquad (3.8)$$

$$Y_{ij}(\phi,\lambda) = \bar{P}_{ij}(\sin\phi)\begin{cases}\cos j\lambda & j \le 0\\ \sin j\lambda & j > 0\end{cases}$$

where ϕ is the latitude, λ the longitude, P_{ij} are the fully normalized Legendre functions, C_{ij} the coefficients, R the mean radius of the Earth and GM the product of the mass of the Earth and the gravitational constant. Then the covariance function is

$$C(P,Q) = \sum_{i=2}^{\infty}\sigma_i\left(\frac{R^2}{rr'}\right)^{i+1} P_i(\cos\psi) \qquad (3.9)$$

with the <u>degree-variances</u>

$$\sigma_i = \left(\frac{GM}{R}\right)^2 \sum_{j=-i}^{i}(\bar{C}_{ij})^2 \qquad (3.10)$$

Here r,r' are the radial distances of P,Q respectively from the origin, and ψ is the spherical distance between P and Q.

Since the coefficients of T at present have only been estimated up to degree and order 360, the degree-variances are represented in parametric form (Tscherning and Rapp, 1974),

$$\sigma_i = \frac{A}{(i-1)(i-2)(i+B)}\left(\frac{R_B}{R}\right)^{2i+2} \qquad (3.11)$$

Here A is a constant, B an integer, and R_B the radius of the so-called Bjerhammar sphere, $R_B < R$.

The choice of this expression has the advantage, that we can determine a mathematical expression for the corresponding norm and thereby determine whether an appropriate smoothing of T is obtained. Also, the covariance function C(P,Q) (and its derivatives) may be represented by closed expressions (Tscherning, 1975).

3.3 Preselected number and type of base functions

In case the selection of base functions is not made implicitly through the collocation condition $L_i(\tilde{T}) = L_i(T)$ and a condition of minimum norm, then the selection must be based on e.g. which kind of source or effect representation is desired. In this case we can distinguish between 3 situations:

- we have less observations than base functions, $n < m$
- we have more observations than base functions, $n > m$ or
- we have the same number, $n = m$.

In the underdetermined case ($n < m$)), the matrix

$$B = \{B_{ij}\} = L_i f_j,\ i=1,\dots,n,\ j=1,\dots,m \qquad (3.12)$$

is investigated. If this matrix has rank n, then it is possible to find (e.g. using singular value decomposition) n linear combinations of the base functions f_i^*, which will serve as new base functions. We are then in the collocation case, n = m, where the coefficients are found by solving n equations with n unknowns,

$$y_i = \sum_{j=1}^{n}\tilde{a}_j L_i(f_j^*)$$

This method is equivalent to using minimum norm collocation in the m dimensional space spanned by the functions f_i^*. They are implicitly being defined as being orthonormal, so the reproducing kernel becomes

$$K(P,Q) = \sum_{i=1}^{m} f_i(P) f_i(Q),$$

see Moritz (1980), eq. (24-8)), and $f_i^* = \Sigma B_{ij} f_j$.

In the overdetermined case, the usual method to get a solution is to use least-squares adjustment, deriving weights from the variance-covariance matrix P^{-1} of the noise vector $\{e_i\}$. Then

$$\sum_{i,j=1}^{n} (y_i - \sum_{k=1}^{m} a_k L_i(f_k)) \{p_{ij}\} \cdot$$

$$\cdot (y_j - \sum_{k=1}^{m} a_k L_j(f_k)) = \min \qquad (3.13)$$

with the solution

$$\{a_j\} = (B^T P^{-1} B)^{-1} B^T P^{-1} \{y\}. \qquad (3.14)$$

When functions f_i are used, which do not go fast to zero compared to the data spacing, then the matrix $(B^T P^{-1} B)$ may be badly conditioned. The problem is then regularized (see e.g. Ilk (1988)) by requiring a simultaneous minimalization of a norm of T, e.g.

$$\alpha \cdot \{a_i\} \{C_{ij}\}^{-1} \{a_i\} \qquad (3.15)$$

where $\{C_{ij}\}$ is a positive definite matrix and α a positive constant. The value of α is determined, so that the minimum of eq. (3.13) is kept as low as possible, but generally α may be selected within a rather large interval, see e.g. Reigber et al. (1987, Section 4).

The solution is obtained by adding $\alpha\, C_{ij}^{-1}$ to the elements of the normal equation matrix in eq. (3.14),

$$\{a_j\} = \{ \alpha \{C_{ij}\}^{-1} + B^T P^{-1} B\}^{-1} (B^T P^{-1}) y \qquad (3.16)$$

It is illustrative again to compare these solutions to the one obtained from least-squares collocation. The vector $y^* = B^T P^{-1} y$ is a new vector of m observations. These observations y^* are, when no regularization is used, reproduced exactly by $\tilde{T}$! We are then back to minimum norm collocation, minimalizing a norm in the space spanned by the functions f_i. This must be a rather strange norm, if not the weights and data distribution is regular, which it however fortunately frequently is! The use of regularization corresponds to the introduction of extra observations of y^* with mean value zero.

The use of as many base functions as observations gives a collocation type solution, since if we do not consider the noise, we have $L_i(\tilde{T}) = L_i(T)$. If the matrix $\{L_i f_j\}$ is symmetric, positive definite, then the norm of $\tilde{T}$ is minimized, for the norm defined as $||\tilde{T}||^2 = \{a_i\} \{B_{ij}\} \{a_j\}$. The inverse $\{B_{ij}\}^{-1}$ then defines implicitly the physical correlations of the observations $\{y_i\}$.

The experience with the use of polynomial interpolation, gives advice on the type of functions which we should use - or avoid. In fact, the theory will lead us to the selection of base functions as in minimum norm collocation, i.e. base functions selected considering the kind of observation functionals we have.

3.4 Selection of approximation method

The above discussed methods are general, in the sense, that they will work with any data type which relationship to T can be expressed using eq. (1.2). This means that we can use SST and SGG data, and also data at the Earth's surface. The data need not to be gridded, or at a fixed altitude.

We see, that we must be able with relative ease to compute quantities like $L_i(f_j)$ or $K(L_i,L_j)$. This has been a deciding factor e.g. for the use of point mass source functions, see e.g. Vermeer (1989a), but for other base functions the necessary expressions, which are only slightly more complex, were already developed (Tscherning, 1975), (Krarup and Tscherning, 1984). However, some modifications are necessary in order to represent the observation functionals in an orbit related frame, cf. eqs. (2.15), (2.16). But even the most complicated function may be tabulated once and for all (see Suenkel (1979)), so that the speed of subsequent computations becomes independent of the kind of expression used.

Several of the above introduced methods have been used already for SGG or SST data or data of a similar type, (Arabelos and Tscherning, 1987), (Ilk, 1988), (Reigber et al., 1987), (Jekeli et al, 1985), (Robbins, 1985), (Tscherning, 1988). The error expression eq. (3.7) has also been used for studies of the influence of sampling rate, instrument noise and satellite altitude, (Tscherning, 1988,1989).

The minimum norm and collocation methods have a solid mathematical basis, and have the possibility for providing error estimates. The weak point is the need for solving large systems of equations.

This numerical effort is controlled in the overdetermined case by the suitable selection of a small number of base functions, thereby implicitly working with weighted data. This could be (and has also been) introduced in the collocation technique (Goldstein and White, 1985) when working with airborne gradiometry. However, one of the important applications of collocation foreseen, is the use of the method for interpolation of the SGG and SST observables, so that data in a regular grid is obtained. This kind of data will then be used as input to the method discussed in the following sections.

4. Integral formula methods

Using the spherical harmonic expansion of T, eq. (3.8) we may establish the direct spectral relationship between T itself and some of the quantities which are explicitly or implicitly observable using SST or SGG. Here one takes advantage of the fact, that the partial derivatives (with respect to Cartesian coordinates) of a harmonic function also are harmonic. Hence T_{rr} is not itself harmonic, but $r^2 \cdot T_{rr}$ is harmonic as can be seen from its spherical harmonic expansion:

$$r^2 T_{rr}(\phi,\lambda,r) =$$

$$= \left(\frac{GM}{r}\right) \sum_{i=2}^{\infty} (i+1)(i+2) \left(\frac{R}{r}\right)^i \sum_{j=-1}^{i} \bar{C}_{ij}\, Y_{ij}(\phi,\lambda) \qquad (4.1)$$

A spherical harmonic analysis of $r^2\, T_{rr}$, r = constant, will therefore give us the coefficients

$$\bar{C}_{ij}{}^* = \frac{GM}{r}(i+1)(i+2)\bar{C}_{ij} \left(\frac{R}{r}\right)^i, \qquad (4.2)$$

so that we obtain for the coefficients of T:

$$\bar{C}_{ij} = \frac{r}{GM}\, \frac{1}{(i+1)(i+2)} \left(\frac{r}{R}\right)^i \bar{C}^*_{ij} \qquad (4.3)$$

The factor r/R is larger than 1, so errors in C^*_{ij} will be amplified for large values of i. This is the cause of the instability inherent in the downward continuation from satellite altitude to the Earth's surface. However, we also divide with (i+1)(i+2), which until a certain degree will stabilize the situation. In Table 4.1 is found the degree i for which $(r/R)^i$ exceeds (i+1)(i+2). It is for the ranges of interest (160 km - 255 km) in the range from 501 to 288. This corresponds to a resolution at the Earth's surface of between 1/4° and 1/2°.

Table 4.1. Degree i for which $(r/R)^i$ exceeds (i+1)*(i+2)

Height Km	i	(i+1)*(i+2)	(r/R)**i	r/R
160	501	252506	255613.3	1.025
165	483	234740	236890.8	1.026
170	466	218556	219212.9	1.027
175	451	204756	208600.3	1.027
180	436	191406	193953.5	1.028
185	422	179352	181324.6	1.029
190	409	168510	170843.1	1.030
195	397	158802	162600.8	1.031
200	385	149382	151918.5	1.031
205	374	141000	143821.8	1.032
210	363	132860	133869.0	1.033
215	353	125670	126648.2	1.034
220	344	119370	122061.5	1.035
225	335	113232	116022.5	1.035
230	326	107256	108766.6	1.036
235	318	102080	104274.4	1.037
240	310	97032	98746.3	1.038
245	302	92112	92370.0	1.038
250	295	87912	88701.5	1.039
255	288	83810	84268.6	1.040

We have

$$\bar{C}_{ij}{}^* = \frac{1}{4\pi} \int\!\!\int r^2\, T_{rr}(\phi,\lambda,r)\, Y_{ij}(\phi,\lambda) \cos\phi\, d\phi d\lambda$$

and then

$$T(\phi',\lambda',r') =$$

$$= \frac{GM}{r'} \sum_{i=2}^{\infty} \left(\frac{R}{r'}\right)^i \sum_{j=-i}^{i} Y_{ij}(\phi',\lambda') \frac{r}{GM}\, \frac{1}{(i+1)(i+2)} \left(\frac{r}{R}\right)^{i+1} \bar{C}^*_{ij} =$$

$$= \frac{1}{4\pi} \int\!\!\int r^2 T_{rr}(\phi,\lambda,r) \sum_{i=2}^{\infty} \frac{1}{(i+1)(i+2)} \left(\frac{r}{r'}\right)^{i+1} \cdot$$

$$\cdot \sum_{j=-1}^{i} Y_{ij}(\phi,\lambda) Y_{ij}(\phi',\lambda') \cos\phi\, d\phi d\lambda =$$

$$= \frac{1}{4\pi} \int\!\!\int r^2 T_{rr}(\phi,\lambda,r) \sum_{i=2}^{\infty} \left(\frac{r}{r'}\right)^{i+1} \frac{(2i+1)}{(i+1)(i+2)} \cdot$$

$$\cdot P_i(\cos\psi) \cos\phi\, d\phi d\lambda \qquad (4.4)$$

where ψ is the spherical distance between the points (ϕ, λ, r) and (ϕ',λ',r').

The function

$$S_{rr}(r,r',\psi) = \sum_{i=2}^{\infty} \left(\frac{r}{r'}\right)^{i+1} \frac{r^2(2i+1)}{(i+1)(i+2)} P_i(\cos\psi) \qquad (4.5)$$

is the integral kernel we need in order to solve the boundary value problem for T, given T_{zz} in space,

$$T(\phi',\lambda',r') = \frac{1}{4\pi} \int r^2\, T_{rr} S_{rr}(r,r',\psi) d\sigma. \qquad (4.6)$$

Unfortunately the sum (4.5) is not convergent for $\psi = 0$, $r > r'$. However, if we are satisfied with obtaining smoothed values of T (mean gravity anomalies or geoid heights) then the kernel could be used, but with the summation only extended up to the values of i given in Table 4.1. (Above this degree, the errors start to be amplified, but another principle for the selection of the summation limit could also be used). The graphs of the kernels corresponding to the heights of 180, 200 and 220 km are shown in Fig. 4.1. The graphs have been normalized, so that the value is equal to 1 for ψ=0. Note that the functions tend to zero quite fast. They could therefore be used for regional recovery of mean geoid heights or mean gravity anomalies.

However, working in a local area, it is possible to use the "flat earth approximation" and work with the Fourier spectrum of T as will be discussed in the following section. These methods also permit that we work with other quantities than T_{rr} and with several quantities simultaneously.

Finally it should be mentioned that attempts have been made to construct kernel functions in the plane similar to the one given by eq. (4.5), see Jekeli (1985, 1986).

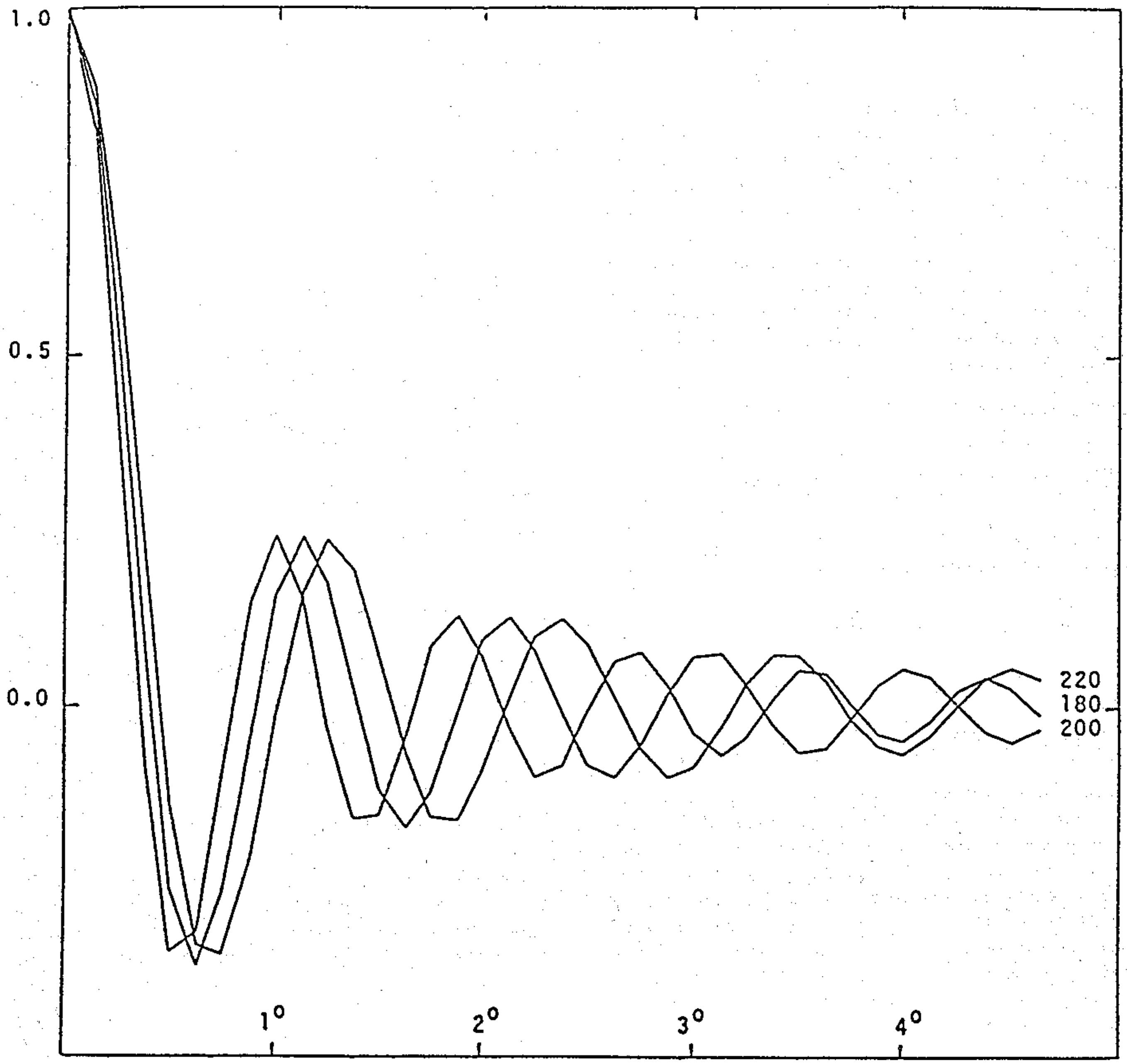

Figure 4.1. The normalized integral kernel S_{rr} for heights 180, 200 and 220 km in the interval from 0° to 5°.

5. Frequency domain or spectral methods

If the observation functionals are orthogonal, i.e. $K(L_i,L_j) = \delta_{ij}$, then the numerical effort in using an approximation method is minimal. This situation happens, if one is able to observe or calculate from the observations the spherical harmonic coefficients C_{ij}.

For regional applications the use of spectral methods in the plane has proven very efficient and given excellent results, see e.g. Kearsley et al. (1985), and (Schwarz et al., 1989). The use of the method for airborne gravity gradiometry has been studied by several authors, and has been reviewed lately by Vassiliou (1986). It is very likely that all the results may be used also for data collected at satellite altitude, especially if the effect of the longer wavelength harmonics are removed, as will be discussed in the next section.

In the following we will use the notation used in Schwarz et al. (1989).

The 2D Fourier transform of a function h(x,y) is defined as

$$H(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{-i(k_x x+k_y y)}dxdy$$

$$:= F\{ h(x,y) \} \qquad (5.1)$$

where k_x, k_y are the wavenumbers corresponding to x, y, respectively and i is the imaginary unit. The function h may be obtained from H using the inverse transform F^{-1}, which is expressed mathematically as eq.(5.1), however with a + sign on the imaginary unit, and the integral divided by $4\pi^2$.

The spatial frequencies u, v in the directions of x, y, respectively are normally used as arguments,

$$k_x = 2\pi u , \qquad k_y = 2\pi v. \qquad (5.2)$$

The Fourier transform has several nice properties which we may take advantage of, see Schwarz et al. (1989) Section 3.2. For us the most important are related to the computation of derivatives

$$F\left[\frac{d^{n+m}}{dx^n dy^m} h(x,y) \right] = (2\pi iu)^n(2\pi iv)^m H(u,v) \qquad (5.3)$$

and the Fourier transform of a convolution,

$$F(h(x,y)*g(x,y)) =$$

$$F(\int\int h(x_o,y_o)g(x_o-x,y_o-y)dx_o dy_o) =$$

$$= H(u,v)G(u,v), \qquad (5.4)$$

with

$$F(g(x,y) = G(u,v) .$$

Since data are only available in a finite area, say a rectangle $-X/2 \le x \le X/2$, $-Y/2 \le y \le \psi/2$, eq.(5.1) can not be used. Instead an estimate of the spectrum can be obtained using the 2D discrete Fourier transform DFT.

Here the integration is performed over the given finite area, and converted into a sum. The data are assumed to be known at the nodes of a regular grid with sampling intervals $\Delta x,\Delta y$. The record lengths may thus be expressed by $X=M\Delta x$, $Y=N\Delta y$, where M and N are the number of points in each direction. Implicitly assuming the data to be periodically extended in the plane, the spectrum becomes discrete with frequency spacings

$$\Delta u = \frac{1}{X} , \Delta v = \frac{1}{Y} .$$

The data sampling interval only allow us to estimation of frequencies up to the Nyquist frequency,

$$u_N = \pm \frac{M}{2} \Delta u , \qquad v_N = \pm \frac{N}{2} \Delta v .$$

The DFT may (approximating the continuous transform) be written as

$$H(m\Delta u, n\Delta v) =$$

$$= \Delta x\Delta y \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} h(k\Delta x,l\Delta y)\cdot e^{-2\pi i(\frac{mk}{M}+\frac{nl}{N})}$$

$$m=0,1,\ldots, M-1 \qquad (5.5)$$
$$n=0,1,\ldots, N-1$$

Here the spectrum is obtained for frequencies from $-u_N$ to $+u_N$ and $-v_N$ to $+v_N$ with the DFT spectrum itself being periodic, with period $2u_N$ and $2v_N$, a consequence of the data sampling. Usually one changes coordinates by a transformation so that $\Delta x = \Delta y = 1$, and used the integer wave numbers k, l and m, n. This gives the basis for using the Fast Fourier Transform algorithm (FFT), the speed of which is a major reason for using spectral methods.

The implicitly assumed periodicity in the space domain produces unavoidable errors near to the boundary. To minimize the problem, windowing may be used, i.e. the function is smoothed out to zero near the boundary, see again Schwarz et al. (1988) for more information.

Suppose the observations of SSG or SGG type have been gridded, and converted into a uniform type of functional relationship, e.g. the second order radial derivative, T_{zz}. Then the spectral relationship may be expressed as (see eq. (5.11)),

$$F(T_{zz}) = F(u,v)\cdot F(T) \qquad (5.6)$$

The inverse Fourier transformed $f(x, y) = F^{-1}(F(u, v))$ is called the transfer function. In this case

$$T_{zz} = f * T .$$

If we want to determine T, it is simply done using

$$T = T_{zz} * f^{-1},$$

where

$$F(f^{-1}) = 1/F(u,v) ,$$

and we must suppose $F(u,v) \neq 0$. Quantities related to T, like geoid heights, gravity anomalies and deflections of the vertical are then easily computed using the transfer functions between these quantities and T, see Schwarz et al. (1989, section 4).

The transfer functions between T and the various second order derivatives at satellite altitude are easily found (see e.g. Vassiliou, 1986). The anomalous potential is at altitude h in planar approximation,

$$T(x,y,h) = \frac{h}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{T(x^1,y^1,0)}{l^3} dx_1 dy_1 \qquad (5.7)$$

with

$$l = ((x-x_1)^2 + (y-y_1)^2 + h^2)^{1/2}.$$

Then

$$F(T(x,y,h)) = \frac{h}{2\pi} F(T(x,y,0))F(l^{-3})$$

$$= F(T(x,y,0)) \cdot e^{-2\pi hq} \qquad (5.8)$$

with $q = (u^2+v^2)^{1/2}$.

The downward continuation is then done using

$$F(T(x,y,0) = F(T(x,y,h)) \cdot e^{2\pi hq}\ , \qquad (5.9)$$

which as we again see has the problem that the higher frequencies are amplified.

The spectra of the first and second order derivatives with respect to z are easily derived by differentiating eq. (4.8) with respect to h,

$$F(T_z(x,y,h)) = -2\pi q\ e^{-2\pi qh} F(T(x,y,0)) \qquad (5.10)$$

$$F(T_{zz}(x,y,h)) = 4\pi^2 q^2\ e^{-2\pi qh} F(T(x,y,0)) \qquad (5.11)$$

The spectral relationships with the derivatives with respect to x and y are also easily derived, and are equal to

$$F(T_{nm}) = F(T) \cdot (i \cdot 2\pi u)^n (i \cdot 2\pi v)^m$$

where n,m is the order of the derivatives with respect to x and y, respectively. (Note, that the spectrum of the Laplace operator is

$$-4\pi^2 u^2 - 4\pi^2 v^2 + 4\pi^2 q^2 = 0$$

as it should be for harmonic functions).

Since it may be feasible, using SGG data, to provide gridded data of a kind being functionally as close as possible to the original data, we must consider what is known as "multiple input-single output" filters, (Vassiliou, 1986, section 3.3). Here the spectrum of T is computed from each of the individual gridded datasets. A weighted mean of the spectra is then computed, using as weights the inverse of the square of the noise of each individual frequency, and contingently also the correlation between the estimated frequency components.

The use of FFT methods has been tested using simulated airborne gradiometer data in a $4^o \times 4^o$ area by (Vassiliou, 1986). Very satisfactory results were obtained both in terms of the recovery of T itself at the ground, but also in terms of computer time. The methods are easily implemented to be used at satellite altitude. Here, naturally rather large areas should be used, considering typical altitudes of 200 km. This could mean that errors due to plane approximation would be significant. However, results by Forsberg and Solheim (1988) for a geoid computation in a $20^o \times 20^o$ area seemed only to be affected very little by such errors.

6. Smoothing and decorrelating the observations

Observations of the low degree and order spherical harmonic coefficients are available, and may be included in a straightforward manner in the regional estimation of the gravity field,

$$\bar{C}_{ij} = L_{ij}(T) = \frac{1}{4\pi} \int_\sigma T \cdot Y_{ij} d\sigma + e_{ij},\ i \le N, \qquad (6.1)$$

where $d\sigma$ is the area element on the unit sphere.

When used within the minimum norm collocation method it gives as a result, that a part of the normal equation matrix (eq. (3.14)) becomes diagonal. It is then easily seen that the use of the observations corresponds to the subtraction of the contribution of the coefficients from the remaining data. The first N terms of the reproducing kernel are modified,

$$K_N(P,Q) = \sum_{i=2}^{N} \sigma_i^e \left[\frac{R}{rr'}\right]^{i+1} P_i(\cos\psi) +$$

$$+ \sum_{i=N+1}^{\infty} \sigma_i \left[\frac{R^2}{rr'}\right]^{i+1} P_i(\cos\psi) \qquad (6.2)$$

where σ_i^e are the error-degree variances,

$$\sigma_i^e = \sum_{j=-i}^{i} \bar{v}_{ij}^{\ 2}, \qquad (6.3)$$

and $\bar{v}_{ij}^2$ is the variance of the error e_{ij} in eq. (6.1).

The variance of all signal quantities,

$$K_N(L_i, L_i)$$

will decrease, see Table 6.1, where the global degree-variance model used in Tscherning and Rapp (1974) has been used. The quantities also become "decorrelated" in the sense, that auto-covariance functions have shorter distances to the first zero points and shorter correlation distance.

On the other hand will certain cross-correlations increase, see Table 6.1, but still the signal variances decrease so much, that the resulting error decreases.

Table 6.1. Standard deviations and correlations of 1/2° mean geoid heights and gravity anomalies, T_{xz} and T_{zz} as a function of altitude and degree N of the OSU81 reference field subtracted. (Spherical distance zero in all cases).

N	St.dev. Geoid (m)	St.dev. Gravity (mgal)	Height (km)	St.dev. T_{zz} E.U.	St.dev. T_{xz} E.U.	Correlation % (T_{zz}, geoid)	Correlation % (T_{zz}, gravity)
0	30.4	33.9	160	0.54	0.37	43	81
36	2.3	27.5	160	0.37	0.26	93	73
72	1.4	24.1	160	0.22	0.15	85	65
108	1.2	22.0	160	0.14	0.10	84	59
0	-	-	200	0.42	0.29	49	79
36	-	-	200	0.25	0.18	94	68
72	-	-	200	0.13	0.09	86	58
108	-	-	200	0.09	0.06	81	46
0	-	-	240	0.34	0.24	55	76
36	-	-	240	0.18	0.12	93	63
72	-	-	240	0.08	0.06	89	53
108	-	-	240	0.06	0.04	81	37

As mentioned in section 3, least squares collocation error estimates may be computed using eq. (3.7). If the leading term K(L,L) (the functional variance) decreases, then the mean square error decreases. This is another way to express the advantage of smoothing the data. And one should note that this smoothing does not mean only loss of information: The effect of the subtracted spherical harmonic expansion will later on be added to the results.

Also the effect of the topography may be subtracted from the data, resulting in a smoothing without loss of information, since the effect will also be added to the result. Here we should naturally be aware, that the spherical harmonic coefficients also include (long wavelength) topographic effects. These effects should therefore be computed from the <u>residual</u> topography, which is the topography referring to a mean height surface with the same resolution as present in the spherical harmonic expansion, see Forsberg and Tscherning (1981) and Denker (1988).

Generally the spherical harmonic coefficients have a 50 - 70 % correlation with the coefficients of the topography for degrees larger than 50, see Rummel et al. (1988). The correlation increases with the degree. Consequently a large part of the gravity field variation will be removed, if we subtract the residual topographic effects. How much is removed depends on the actual area, see e.g. Forsberg (1984).

A consequence of the removal of the major part of the long and very short wavelength gravity field variation, is a field which is statistically much more homogeneous, see Forsberg (1986). It will be possible to use the same empirical covariance function for large areas. As a further consequence, error estimates will be locally much more reliable.

Gross errors in the data will also be easier to detect, since the signal to noise ratio increases. The estimation of parameters (instrument biases, state vectors) will also be facilitated, since their effect should stand out more clearly.

It is naturally an extra burden to compute all these effects when processing SST or SGG data. However, the contributions from the spherical harmonic expansion may be computed very fast in the nodes of a regular grid, from which the values in an arbitrary point are easily computed by interpolation. The topographic effects may also be computed in a grid, contingently using FFT (see Schwarz et al., 1989). The values in an arbitrary point are then obtained by fast interpolation in the grid.

Finally one further possibility for smoothing the field should be mentioned: In areas with a good gravity coverage of large extent it is possible to compute the SST or SSG quantities with very good results. Consequently only data noise and biases are left. This may be used to study the nature of the data noise (its statistical properties) and identify the important bias (or tilt) parameters. Such a "calibration" area should naturally, if possible, be selected where the gravity field is as smooth as possible.

7. <u>Conclusion</u>

Many methods are available for the regional modelling of the anomalous gravity field using SGG or SST data. Approximation methods exist, which may be used to grid the data, which then may be treated using frequency domain methods, globally or regionally using FFT. The methods should be compared both with respect to the quality of the results and with respect to ease of use in terms of computer resources and data management effort. It is a clear advantage to remove the effect of the best possible spherical harmonic expansion and the effect of the residual topography from the data. This effect could be precomputed in the nodes of a grid at satellite altitude, which then could be used to determine the needed quantities along the satellite orbit.

The regional use of approximation methods, which use fewer base functions than observations, must be done with great care. Their use corresponds to the use of weighted or averaged data, and one must be sure which kind of averages are found, so that as little information as possible is lost. Alternatively, averaged observations should be computed (the template method), and used within the framework of least squares collocation. This gives the possibility for computing error estimates, which are very important for the later use of the estimated quantities for e.g. geophysical investigations.

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