Lecture 4. **Global Gravity field-modelling**.

Before 1990 Earth's surface not known precisely: now known from radar altimetri, InSAR and GPS

Before 1990: Problem formulated as Boundary-value problem for elliptic partial differential-equation, boundary S, for volume v.

$$\Delta W = 2\omega^2 = \sum_{i=1}^{3} \frac{\partial^2 W}{\partial X_i^{\,2}}$$

Solution, if gravity vector component $\left.\frac{\partial W}{\partial n}\right|_S$ **vertical to surface, S, is known.**

Gravity field modelling

May be converted to integral-equation, l= distance:

$$-2\pi W+\iint_S\left(W\frac{\partial}{\partial n_S}\left(\frac{1}{l}\right)-\frac{1}{l}\frac{\partial W}{\partial n_S}\right)dS$$

$$+2\pi\omega^2(X^2+Y^2)+2\omega^2\iiint_v\frac{dv}{l}=0$$

Gravity modelling. Linearisation.

Better to use anomalous potential T=W-U and an approximate Earth-surface, Telluroid Σ determined by the points Q on the ellisoidal-normal:

$$
-2\pi T + \iint_{\Sigma} \left(T \frac{\partial}{\partial n_{\Sigma}} \left(\frac{1}{l} \right) - \frac{1}{l} \frac{\partial T}{\partial n_{\Sigma}} \right) d\Sigma
$$

$$
and\ \Delta T = 0
$$

Because same Centrifugal potential for U and W + 0’ og 1’ orden terms.

Gravity field modelling.

Anomalous gravity field:

T=W-U,

Observations are linear functionals:

$$\delta g \approx -\frac{\partial T}{\partial r}\Big|_P \; or \; -\frac{\partial T}{\partial h}\Big|_P \; (better\ approximation)$$

$$\Delta g \approx -\frac{\partial T}{\partial r}\Big|_P + \frac{T}{\gamma}\left(\frac{\partial \gamma}{\partial n}\right) \approx -\frac{\partial T}{\partial r}\Big|_P - \frac{T(P)}{r}$$

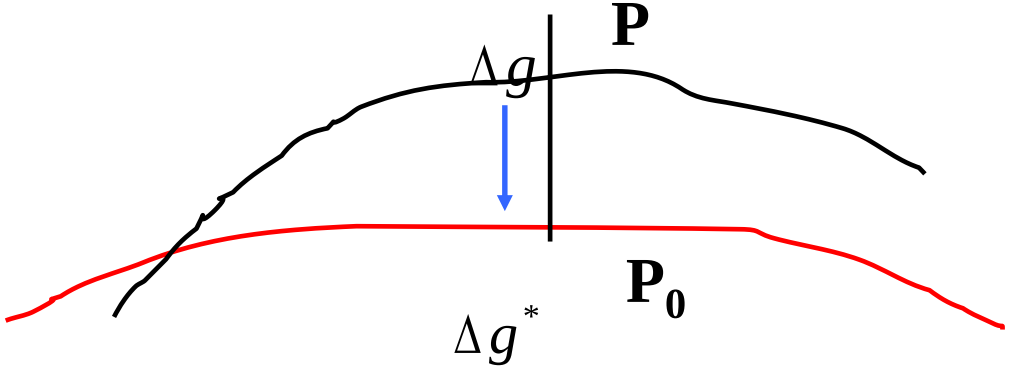
$$\zeta = \frac{T(P)}{\gamma(Q)}$$

Gravity field modelling on Sphere or Ellipsoid.

If Earth may be regarded as spherical or ellipsoidal, then Laplace-equation may be solved using different methods.

Data must be moved from Earth’s surface to the Sphere or Ellipsoid and

Masses between surface and Sphere/Ellipsoid must be removed or data corresponding to harmonic function outside sphere/ellipsoid must be determined.



Coefficient-comparison.

Example: polynomium:

$$p(t) = a_0 + a_1 t + a_2 t^2$$

$$p'(t) = \quad a_1 + 2a_2 t$$

If we know that a_0=0, then p(t) can be found !

For anomalous potential are 4 first coefficients = 0, because we know GM and Earth-center.

Gravity field modelling, Sphere.

$$T(r,\varphi,\lambda) = \sum_{i=2}^{\infty} \frac{GM}{r}\left(\frac{R}{r}\right)^{i} \sum_{j=-i}^{i} \overline{P}_{ij}(\sin\varphi)\cdot\begin{Bmatrix}\cos j\lambda \\ \sin|j|\lambda\end{Bmatrix}\Delta\overline{C}_{ij}$$

a put equal to R (mean radius), then

$$r = R + h, \quad \overline{\varphi} = \varphi$$

$$\Delta g(R,\varphi,\lambda) = \sum_{i=2}^{\infty} \frac{GM}{R}\frac{i-1}{R} \sum_{j=-i}^{i} \overline{P}_{ij}(\sin\varphi)\cdot\begin{Bmatrix}\cos j\lambda \\ \sin|j|\lambda\end{Bmatrix}\Delta\overline{C}_{ij}$$

$$= \sum_{i=2}^{\infty} \frac{GM}{R} \sum_{j=-i}^{i} \overline{P}_{ij}(\sin\varphi)\cdot\begin{Bmatrix}\cos j\lambda \\ \sin|j|\lambda\end{Bmatrix} H_{ij}$$

$$\Rightarrow \Delta\overline{C}_{ij} = H_{ij}\frac{R}{i-1}$$

Gravity field-modelleing.

Spherical harmonic analysis gives H_{ij} and thereby $\overline{C}_{ij}$

Normalized Legendre-functions used, so that we get orthonormal-system Y_{ij} in space of square-integrable functions $L_2(S)$

$$\frac{1}{4\pi}\iint_S \overline{P}_{ij}(\sin\varphi)^2 \begin{Bmatrix} \cos^2 j\lambda \\ \sin^2 j\lambda \end{Bmatrix} \cos\varphi d\varphi d\lambda = 1$$

$$Y_{ij}(\varphi,\lambda) = \overline{P}_{ij}(\sin\varphi)\begin{Bmatrix} \cos j\lambda \\ \sin|j|\lambda \end{Bmatrix} \begin{cases} j \geq 0 \\ j < 0 \end{cases}$$

Gravity field modelling.

We use that the coefficients in the development in the space may be determined by calculating the inner product (integral over sphere) of basis-functions with the functions:

$$\frac{GM}{R}\Delta\overline{C}_{ij} = \frac{1}{4\pi}\iint_S T(\varphi',\lambda',R)\cdot Y_{ij}(\varphi',\lambda')dS$$

$$(i-1)\frac{GM}{R^2}\Delta\overline{C}_{ij} = \frac{1}{4\pi}\iint_S \Delta g(\varphi',\lambda',R)\cdot Y_{ij}(\varphi',\lambda')dS$$

Gravity field modelling, Ellipsoid.

$$T(r,\varphi,\lambda) = \sum_{i=2}^{\infty} \frac{GM}{r}\left(\frac{a}{r}\right)^{i} \sum_{j=-i}^{i} \overline{P}_{ij}(\sin\varphi)\cdot\begin{Bmatrix}\cos j\lambda \\ \sin|j|\lambda\end{Bmatrix}\Delta\,\overline{C}_{ij}$$

Then integration over ellipsoid:

$$\Delta\overline{C}_{ij} = \frac{1}{4\pi a\gamma}\sum_{n=0}^{N-1} r_n^e \sum_{k=0}^{s'} \frac{L_{ijk}}{\overline{S}_{n-2k,|j|}(b/E)} \;\; \frac{\overline{IP}^{\,n}_{n-2k,|j|}}{(i-2k-1)q^{i}_{i-2k}}$$

$$\sum_{n=0}^{2N-1}\overline{\Delta\overline{g}}^{e}_{nm}\begin{Bmatrix}IC \\ IS\end{Bmatrix}^{n}_{j} \quad \begin{matrix} j \geq 0 \\ j < 0 \end{matrix}$$

$$\overline{IP}^{\,n}_{i,|j|} = \int_{\delta_n}^{\delta_{n+1}} \overline{P}_{i|j|}(\cos\delta)\sin\delta d\delta$$

$$\begin{Bmatrix}IC \\ IS\end{Bmatrix}^{n}_{j} = \int_{\lambda_m}^{\lambda_{m+1}} \begin{Bmatrix}\cos m\lambda \\ \sin|m|\lambda\end{Bmatrix} d\lambda$$