Lecture 3. **Gravity field observation methods.**

1. Plumb-line direction: Astronomical latitude and longitude (direction of gravity vector).
2. Magnitude of gravity,
3. Acceleration and velocity (vectors) of body in free fall (satellite)
3. Gravity potentials 2.'orden partial derivatives or first order derivatives of gravity vector
4. Tidal-effects, (on Earth and in satellite)
5. Potential-differences (Precision or hydrostatic levelling), and variations of kinetic energy.

Astronomical measurements

Star-coordinates from star-catalogue:

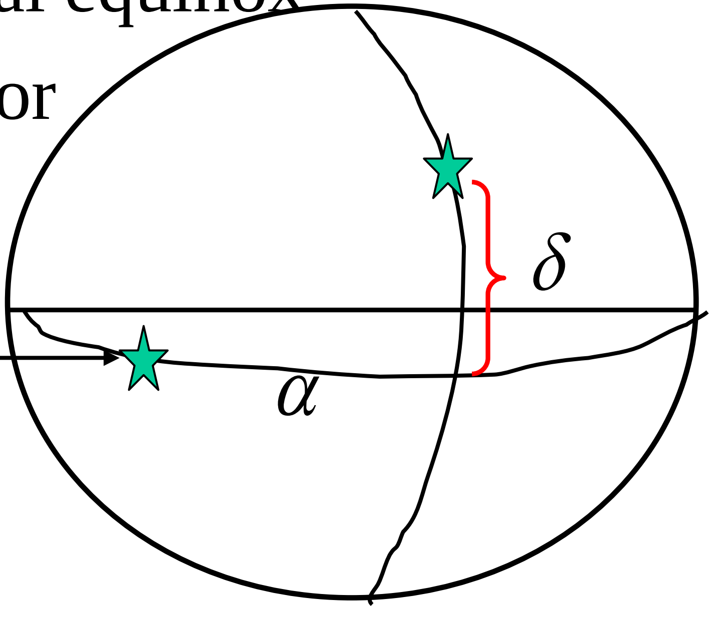
$$(\alpha, \delta, parallax, eigen - movement)$$

Refer to ekliptika

α longitude from vernal equinox

δ latitude from Equator

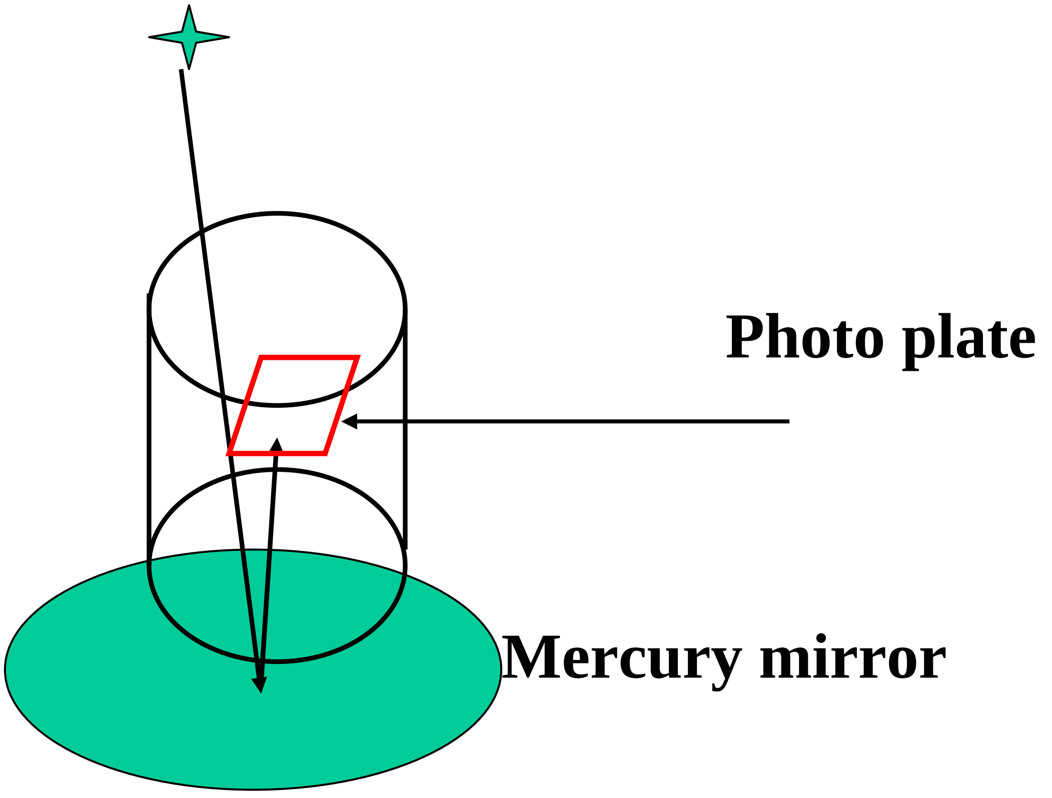
Vernal equinox



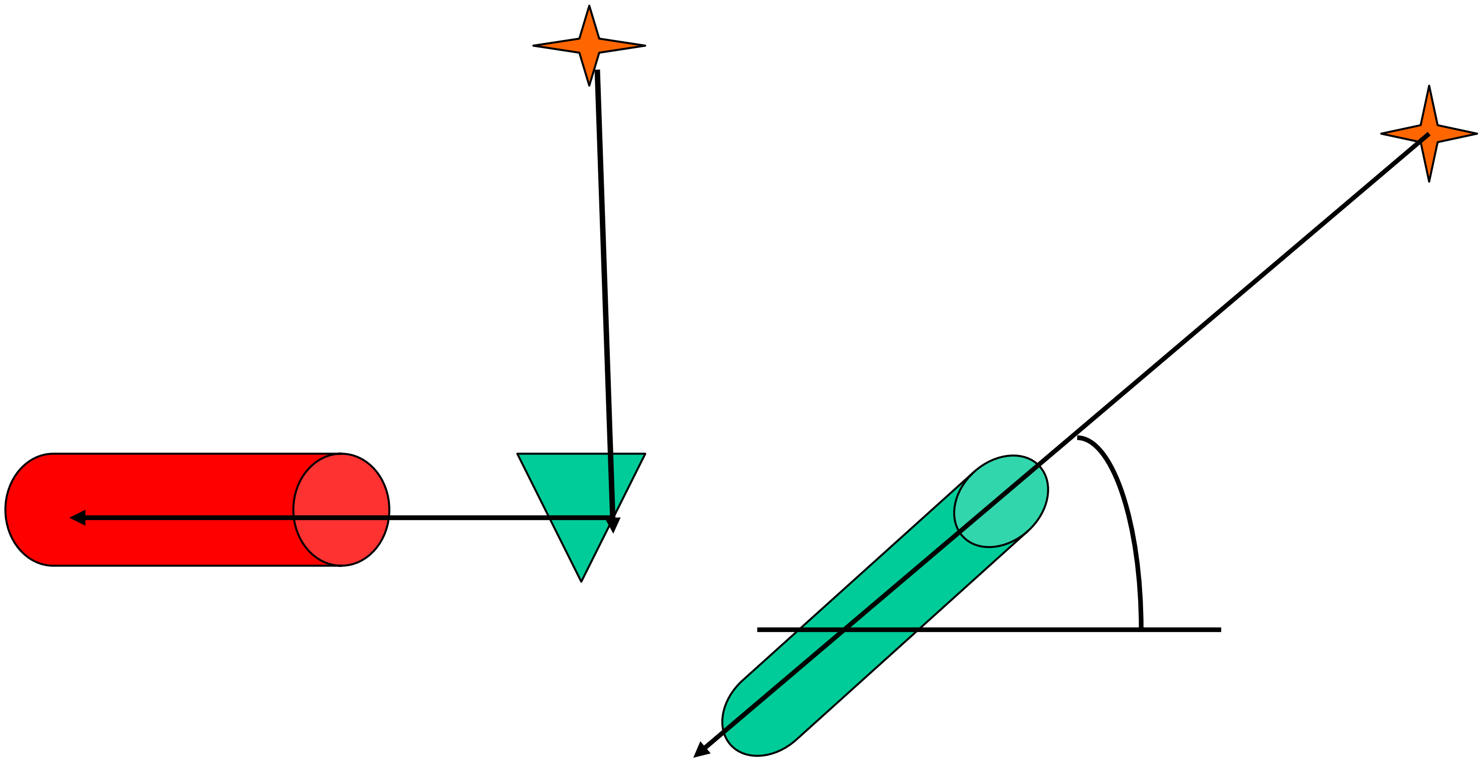
Zenith-camera.

Positioned in level, so refraction is small.

Stars, symmetric about Zenith to be used.

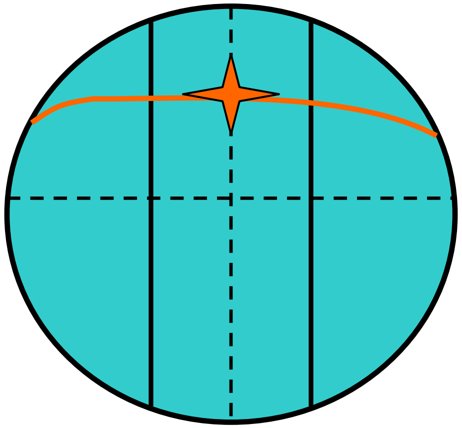


Prisme-astrolabium and theodolite.



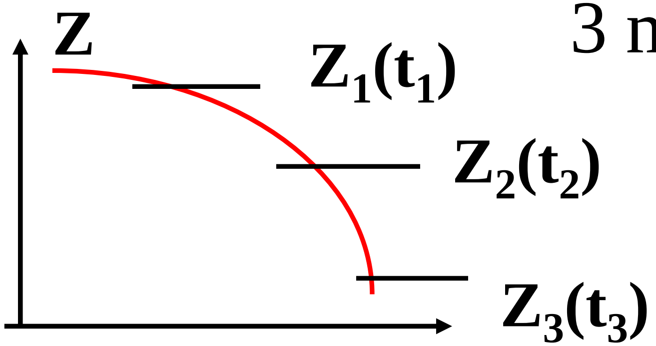
Observations in the local meridian-plane.

Time gives longitude
Height-angle gives latitude



Gravity from Free fall. Vakuum, Seismic protected.

Free fall: **Height measured as a function of time**.



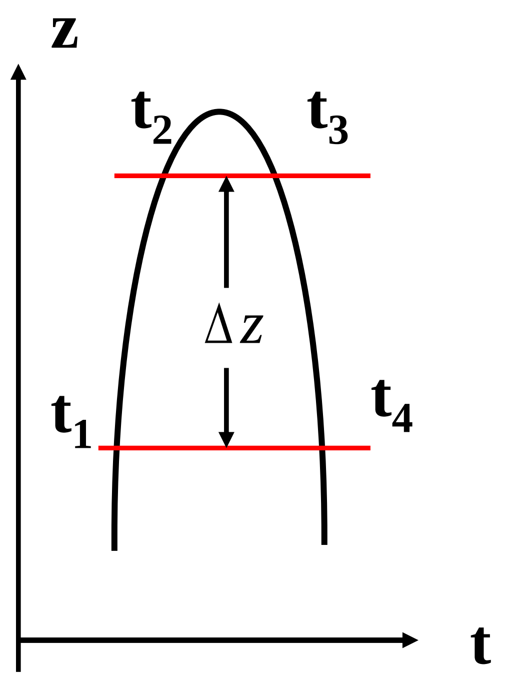
3 measurements, eliminate z_0, $\dot{z}_0$

$$z = z_0 + \dot{z}_0 t + \frac{g}{2} t^2$$

$$g = 2 \frac{(z_3 - z_1)(t_2 - t_1) - (z_2 - z_1)(t_3 - t_1)}{(t_3 - t_1)(t2 - t_1)(t_3 - t_2)}$$

Gravity measurements, throw, vakuum, seismic protected.

60 measurements over 0.5 m gives $\pm\, 0.1 \mu m / s^2$



$$g = \frac{8 \Delta z}{(t_4 - t_1)^2 - (t_3 - t_2)^2}$$

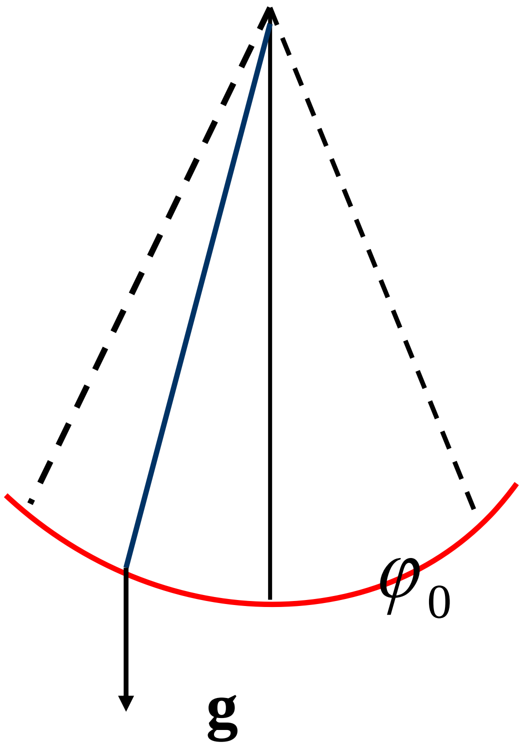
Gravity: Pendulum.

1.Matematical Pendulum: point-mass, weightless thread.

$$l\ddot{\varphi} + g\sin\varphi = 0$$

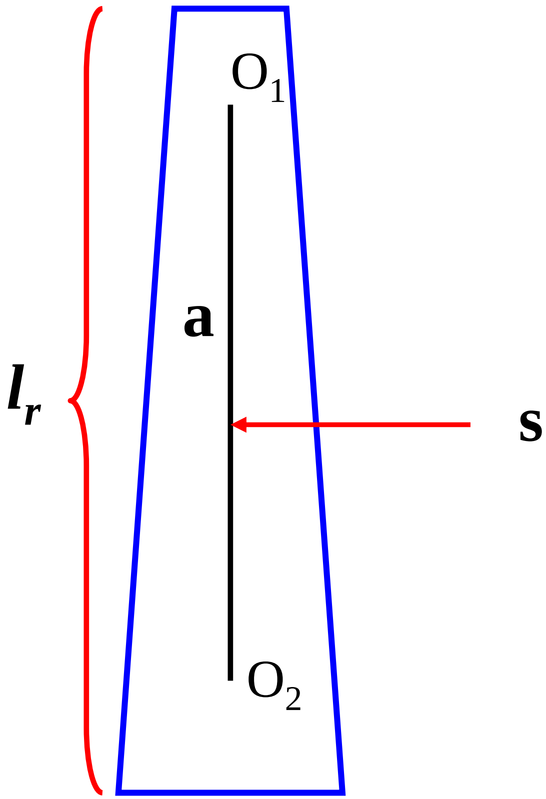
Period:

$$T = 2\pi\sqrt{\frac{g}{l}}\left(1 + \frac{\varphi_0^2}{16} + \ldots\right), \varphi_0 < 30'$$



.   ”Real” pendulum.

Same equations, but l becomes



$$l_r = \frac{J}{m \cdot a}$$

O_1

J=Inertimoment wrt s.

l_r: Distance between points O_1 and O_2 where ocillation times are equal.

l typially 0.25 - 1.0 m

Precision: $\pm 3\mu m / s^2 = 0.3 \quad mgal$

Pendulum at Potsdam, Germany..

Time of occillation
depends on gravity.

Relative Gravity-measurements: Spring-gravimeter.

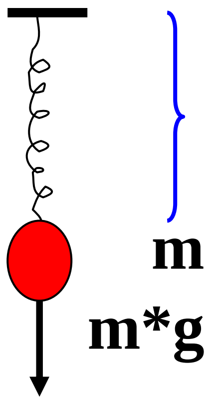
Hook's Law: extension of spring proportional to force.

$$mg - f(l - l_0) = 0$$

l_0: *Spring without weight*

f: *spring – constant.*

Small changes:



,

$$\frac{\Delta g}{\Delta l} = \frac{f}{m} = \frac{g}{l - l_0} \approx \frac{10}{l - l_0}$$

$$\Delta g \approx 10^{-8} \Rightarrow \Delta l \quad for\ l = 0.1 \pm 1nm$$

Gravity measurements with Rotating system.

Lever Spring Balance:

$$mga\cos\alpha = \tau(\alpha_0 - \alpha)$$

a: *leverlength*

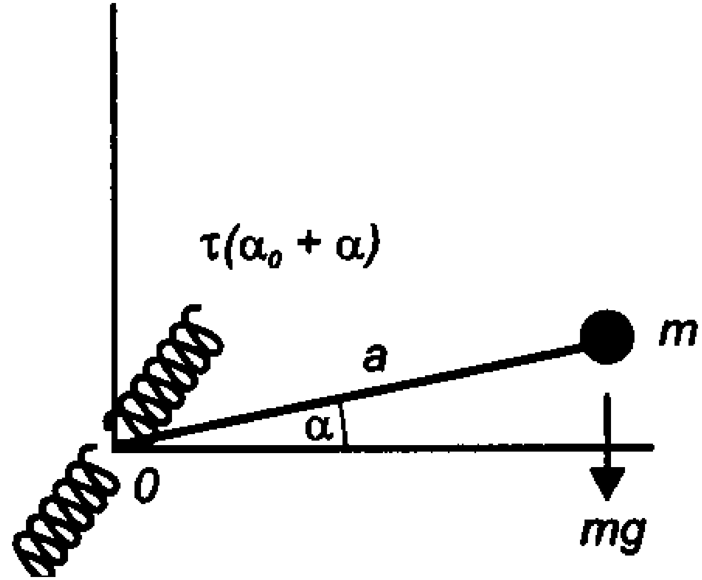
α: *angle between lever and* 0

α_0: *pretension angle of sprin*

τ: *torsion* constant

$If \quad \alpha \to 0 \Rightarrow$

$$\Delta g = \frac{\tau}{ma}\Delta\alpha = \frac{g}{\alpha_0}\Delta\alpha$$



General lever spring balance.

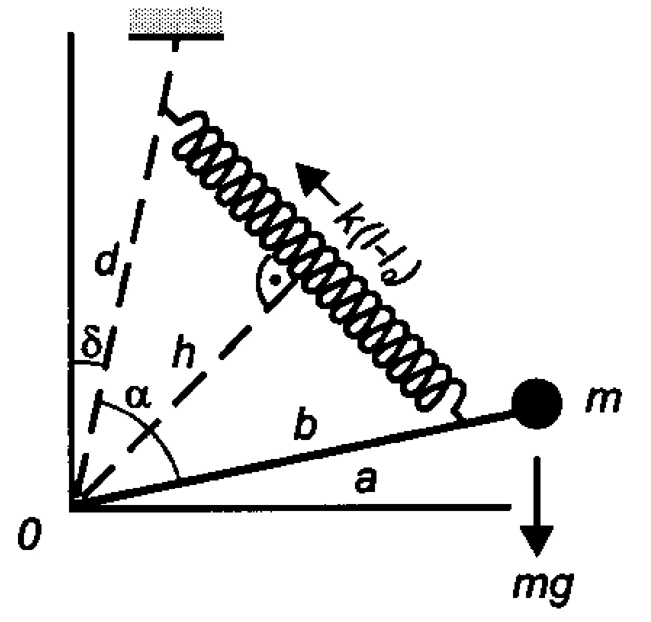
Equilibrium condition:

$$h = (bd\ /\ l)\sin\alpha$$

$$mg\sin(\alpha+\delta) - f\cdot b\cdot d\cdot\frac{l-l_0}{l}\sin\alpha = 0$$

$l_0 = 0,$ *gives sensitivity*:

$$\frac{d\alpha}{dg} = \frac{\sin(\alpha+\delta)}{g\sin\delta}$$



Lacoste-Romberg gravimeter.

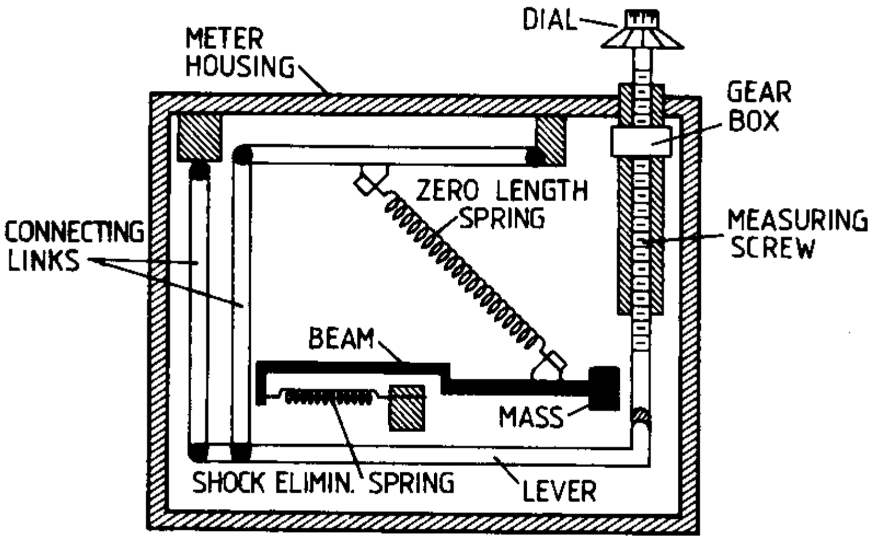
Astatizing: l_0=0 (0-spring)

$$\delta\ small,\ \alpha \approx 90^0$$

$$a = 0.1\,m,\ \ \alpha + \delta = 90^o,$$

$$\delta \approx 100''$$

$$Displacement: \quad \pm 2\,\mu m.$$



Sensitivity 2000 x larger !!

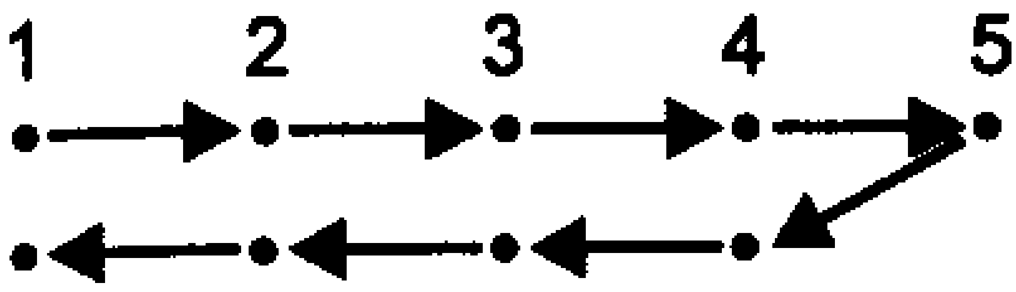
Relative gravity measurements. Controle.

Measurements on known points:

Several instruments are used. (Jump in spring may be identificed).

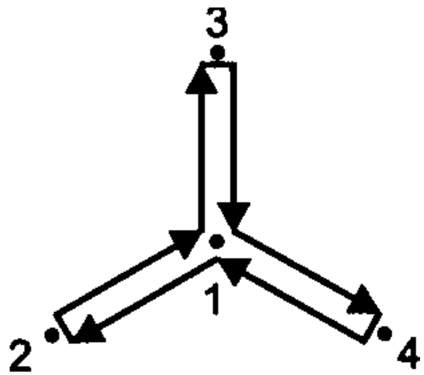
Measurements must be repeated:

Profile:

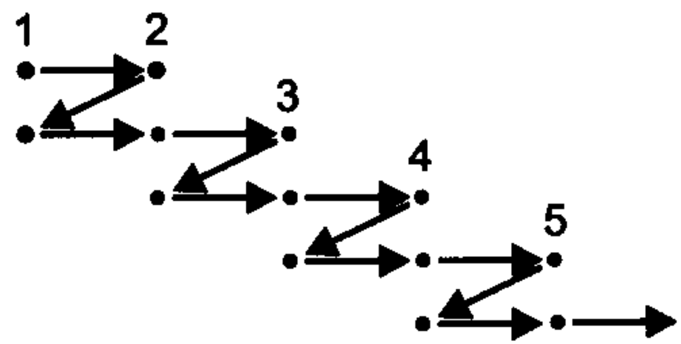


Relative gravity-measurements. Control.

Star:



Steps:



If we measure on calibration-points, calibration curve may be determined.

Relative gravity measurements at sea or in the air.

Gyrostabilised board necessary

Spring limited to only move in one plane (direction of movement)

We must take into account the Eötvös effect – which may be determined from position and velocity.

$$g' = \ddot{r}' - R_b^l f^b + (2\omega_{ie}' + \omega_{el}') \times \dot{r}'$$

Skalar for gyro close to Earth

$$: \delta g_{E\ddot{o}t} = 40 \cos\varphi \sin\alpha + 0.012 v^2 \ \mu m s^{-2}$$

$(v \quad in\ km/h)$

Gravity databases.

Globally: Bureau Gravimetrique Internationale

http://bgi.cnes.fr:8110/bgi_f.html

NGA – USA,

Denmark: KMS, DTU-Space. Gravity from Altimetry.

Gravity-data-standard GRAVSOFT format:

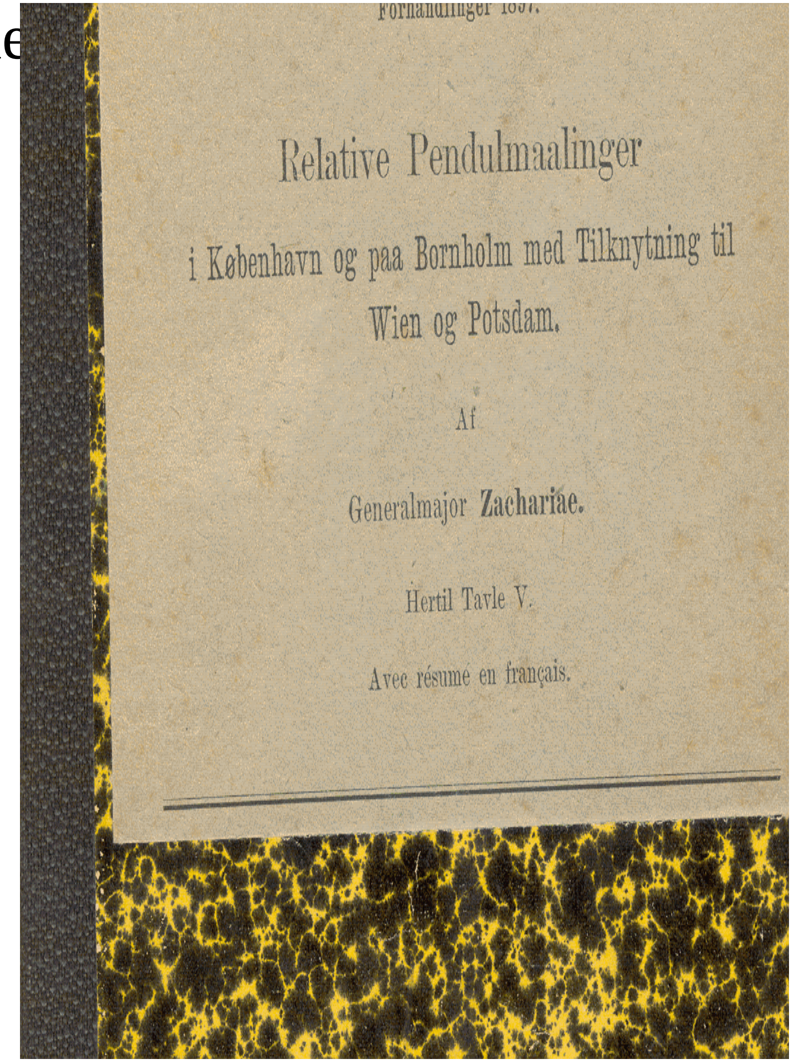
Latitude, longitude in decimal degrees

height above sea or depth, gravity, gravity anomaly.

Source-code, correction to IGSN71 (if necessary).

Publications: Geodætisk Instituts skrifter, KMS Technical reports.

Gravity measureme



Forhandlinger 1897.

Relative Pendulmaalinger

i København og paa Bornholm med Tilknytning til Wien og Potsdam.

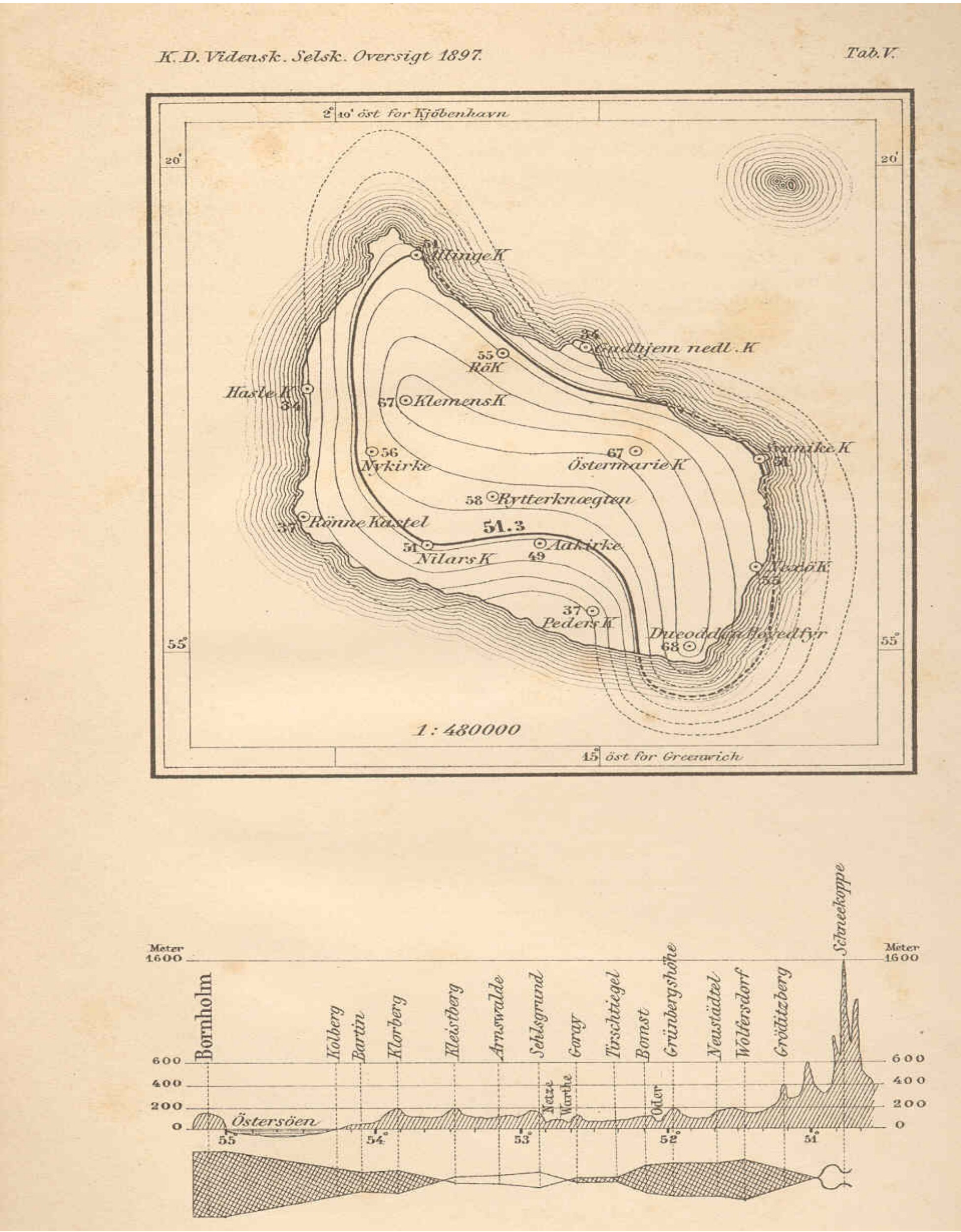
Af

Generalmajor **Zachariae.**

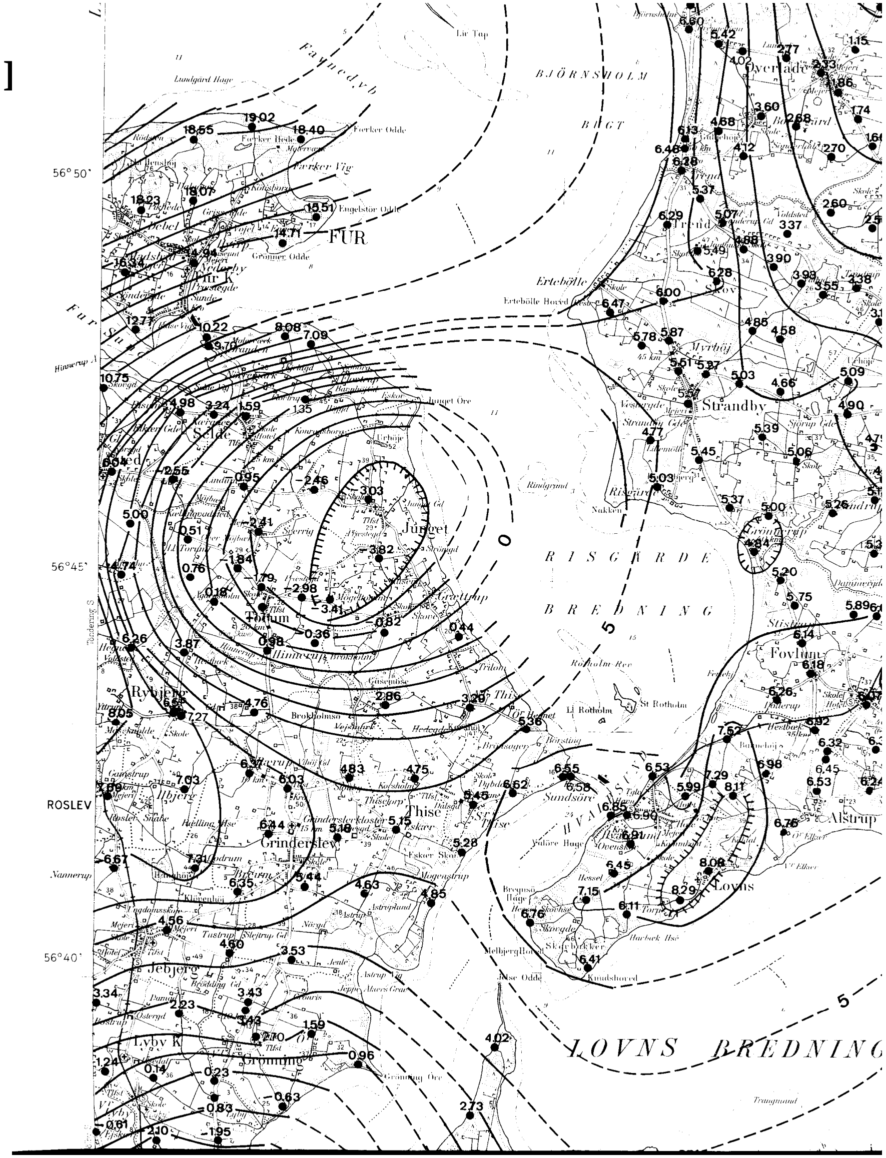
Hertil Tavle V.

Avec résumé en français.

Gravity measur



Gravity



56°50'
56°45'
56°40'
ROSLEV
FUR
BJÖRNSHOLM
BUGT
Strandby
Erteböllc
Myrhöj
Fovlum
RISGÅRDE
BREDNING
Junget
Thise
Grinderslev
Jebjerg
Lyby K
Lovns
Alstrup
Sundsöre
LOVNS BREDNING
St Rotholm
0
5

Gravity data, Saxov.

TABLE 9.

Principal data for detail stations of the Worden gravimeter network in Central Jylland.

Number of Station	Longitude East of Greenwich	Northern Latitude	Height in metres	Normal Gravity in Gals	Observed Gravity in Gals	Bouguer Correction in mGals	Bouguer Anomaly in mGals
	λ	φ	H	γ_0	g	$B.C.$	Δg
M 1111							
1.	09°44'.18	57°02'.21	14.05	981.68527	981.67605	3.16	− 6.06
2.	44.77	02.20	6.37	68525	67762	1.43	− 6.20
3.	43.30	02.16	26.05	68520	67383	5.86	− 5.51
4.	42.73	01.92	19.33	68487	67544	4.35	− 5.08
5.	40.86	01.71	2.66	68458	68040	0.60	− 3.58
M 1112							
1.	09°53'.44	57°03'.10	3.12	981.68649	981.67819	0.70	− 7.60
2.	52.41	02.95	2.51	68629	67804	0.56	− 7.68
3.	47.63	02.62	1.26	68583	67733	0.28	− 8.22
4.	49.04	02.60	0.94	68581	67718	0.21	− 8.41
5.	46.03	02.45	5.02	68560	67726	1.13	− 7.21
6.	52.55	02.38	14.00	68550	67411	3.15	− 8.24
7.	51.32	02.32	1.97	68542	67619	0.44	− 8.79
8.	46.88	02.29	3.82	68538	67686	0.86	− 7.66
9.	50.63	02.24	1.82	68531	67627	0.41	− 8.63
10.	48.20	02.08	2.80	68509	67681	0.63	− 7.65
11.	53.59	02.07	29.19	68508	67104	6.56	− 7.47
12.	48.87	02.03	2.43	68502	67656	0.55	− 7.91
13.	52.67	01.84	15.74	68476	67282	3.54	− 8.40
14.	53.82	01.80	55.31	68470	66394	12.44	− 8.33
15.	48.04	01.78	3.77	68468	67689	0.85	− 6.94
M 1113							
1.	10°01'.51	57°04'.28	1.57	981.68812	981.68461	0.35	− 3.15
2.	09 58.90	04.08	1.35	68784	68320	0.30	− 4.34
3.	58.59	03.87	1.87	68755	68249	0.42	− 4.64
4.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26.	09°56'.79	57°01'.92	38.58	981.68487	981.66975	8.68	− 6.44
27.	58.53	01.81	29.77	68472	67239	6.70	− 5.63
M 1114							
1.	10°05'.26	57°01'.84	2.83	981.68476	981.68255	0.64	− 1.57
M 1208							
1.	09°17'.29	56°58'.95	3.51	981.68078	981.68177	0.79	+ 1.78
2.	14.95	58.15	1.29	67968	68430	0.29	+ 4.91
3.	17.14	58.10	7.35	67961	67919	1.65	+ 1.23
4.	16.25	58.06	4.19	67955	68136	0.94	+ 2.75
5.	16.57	57.66	8.57	67900	67859	1.93	+ 1.51
6.	14.87	57.60	17.98	67892	67980	4.04	+ 4.92
7.	15.22	57.57	23.85	67888	67772	5.36	+ 4.20
8.	15.85	57.46	28.89	67873	67522	6.50	+ 2.99
M 1209							
1.	09°25'.94	57°00'.65	1.26	981.68312	981.68446	0.28	+ 1.62
2.	25.25	00.50	1.30	68291	68418	0.29	+ 1.56
3.	24.56	00.38	0.94	68275	68442	0.21	+ 1.88
4.	23.07	00.09	1.36	68235	68406	0.31	+ 2.02
5.	21.76	56 59.94	1.73	68214	68323	0.39	+ 1.48
6.	17.86	59.93	4.93	68213	68303	1.11	+ 2.01
7.	20.58	59.81	1.48	68196	68292	0.33	+ 1.29
8.	26.07	59.68	5.91	68179	68353	1.33	+ 3.07
9.	19.51	59.55	3.31	68161	68206	0.74	+ 1.20
10.	26.43	59.52	15.14	68157	68148	3.40	+ 3.32
11.	25.45	59.47	5.28	68150	68364	1.19	+ 3.33
12.	23.32	59.45	1.89	68147	68339	0.43	+ 2.35
13.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]