Lecture 2. Gravity-dependent coordinates

Astronomical system: Φ_A, Λ_A

astronomial longitude and latitude.

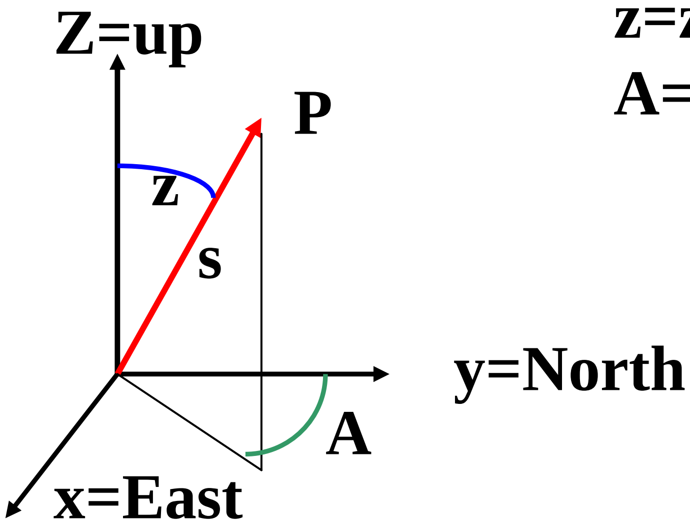
W: value of potential, g= $|\nabla W|$

$$\mathbf{g} = \nabla \mathbf{W} = -\mathbf{g} \cdot \mathbf{n} = -g \begin{Bmatrix} \cos\Phi_A \cos\Lambda_A \\ \cos\Phi_A \sin\Lambda_A \\ \sin\Phi_A \end{Bmatrix}$$

$$\Phi_A = \arctan \frac{-W_z}{\sqrt{W_x^2 + W_y^2}}, \Lambda_A = \arctan \frac{W_y}{W_x}$$

Local astronomical system

Plumb-line-oriented.



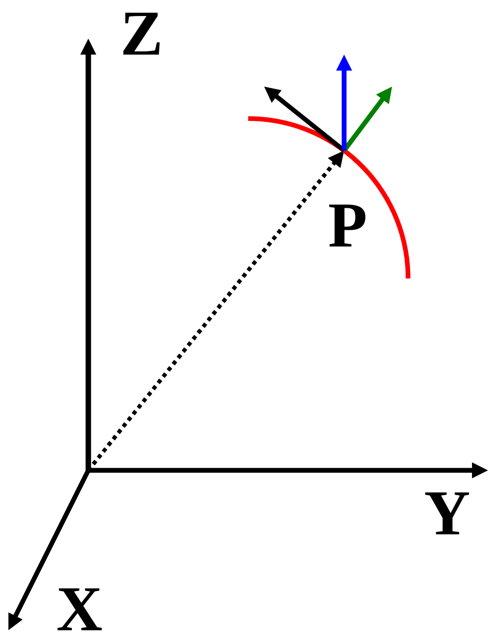
z=zenith-distance

A=azimuth,

positive with the clock, from north

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = s \begin{Bmatrix} \sin z \sin A \\ \sin z \cos A \\ \cos z \end{Bmatrix}$$

Local astronomical system II,



y - North

x - East

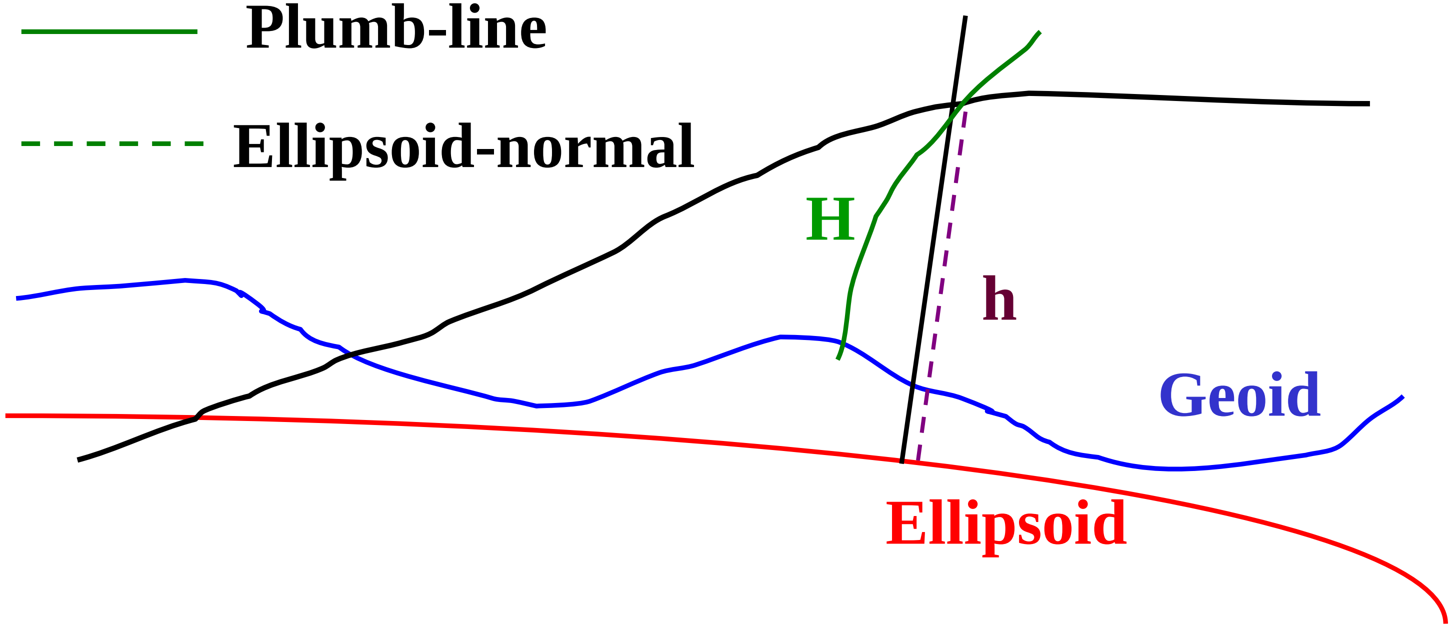
z - Up

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = A^{-1} \begin{Bmatrix} X - X_P \\ Y - Y_P \\ Z - Z_P \end{Bmatrix}$$

$$A^{-1} = \begin{Bmatrix} -\sin\Lambda & \cos\Lambda & 0 \\ -\sin\Phi\cos\Lambda & -\sin\Phi\sin\Lambda & \cos\Phi \\ \cos\Phi\cos\Lambda & \cos\Phi\sin\Lambda & \sin\Phi \end{Bmatrix}$$

The Geoide as a reference-surface.

As altitude must be used C= $W_0 - W_P = -\int_0^P dW = \int g\,dn$



Geopotential number: Unit gpu, 100 m^2/s^2=kgal x m

Normal-potential, U.

Approximation U to W, which

(1) – represent the “normal” gravity variation as a function of latitude and altitude.

(2) - T=U-W, anomalous potential, enhances geophysically interesting mass anomalies.

(3) - U generated of a “nice” mass-distribution with correct GM

(4) – has an equipotential surface $U=U_0$, which is the ellipsoid.

U expressed in Ellipsoidic Harmonic Functions

•

$$U(u,\theta,\lambda) = \Phi(u,\theta) +$$

$$GM\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{Q_{n|m|}(i\frac{u}{E})}{Q_{n|m|}(i\frac{b}{E})}$$

$$\cdot\, C_{nm}P_{n|m|}(\cos\theta)\begin{cases}\cos m\lambda, m\geq 0\\ \sin|m|\lambda, m<0\end{cases}$$

The Normal-potential

Since the ellipsoid must be an equipotential- surface:

(1) - C_{nm}=0, m ≠ 0.

(2) - symmetry about Equator, so C_{nm}=0, m odd

(3) - U generated of a nice masse-distribution with correct GM and centrifugal-potential

(4) -

$$\Phi(\theta) = \frac{1}{2}\omega^2(\mathbf{u}^2 + \mathbf{E}^2)\sin\theta^2 =$$

$$\frac{1}{2}\omega^2(\mathbf{u}^2 + \mathbf{E}^2)\frac{2}{3}(1 - \mathbf{P}_2(\cos\theta))$$

The Normal-potential

Centrifugal-potentialet must balance the gravity potential on the ellipsoid

$$U = \frac{GM}{E} \arctan \frac{E}{u} + \frac{\omega^2}{2} a^2 \frac{q}{q_0} \left(\sin\beta^2 - \frac{1}{3} \right)$$

$$+ \frac{\omega^2}{2} (u^2 + E^2) \cos\beta^2 ,$$

$$q = \frac{1}{2} ((1 - 3 \frac{u^2}{E^2} \arctan\left(\frac{E}{b} \right) - 3 \frac{u}{E}), q_0 = q(u = b)$$

The Normal-potential

On the ellipsoid, u=b

$$U(\beta,\lambda,b) =$$

$$\frac{GM}{E}\arctan(\frac{E}{b}) + \frac{\omega^2}{2}a^2(\sin\beta^2 - \frac{1}{3}) + \frac{\omega^2}{2}a^2\cos\beta^2$$

$$\sin ce \quad \sin\beta^2 + \cos\beta^2 = 1$$

$$U = \frac{GM}{E}\arctan(\frac{E}{b}) + \frac{\omega^2}{2}a^2\frac{2}{3}, cons\tan t!$$

The Normal-gravity.

Normal gravity on Equator: γ_a

…………………….. Poles : γ_b

Pizetti showed: $$2\frac{\gamma_a}{a} + \frac{\gamma_b}{b} = \frac{3GM}{a^2 b} - 2\omega^2$$

Clairout: $$\beta = \frac{\gamma_b - \gamma_a}{\gamma_a} \quad gravity - flattening$$

$$f + \beta = F(\omega, a, b, \gamma_a)$$

Geometry/gravity

Show interconnection between the flattening of the Earth and the change of gravity.

Or from $\gamma_a, \gamma_b, \omega, a$

Is it possible (iteratively) to find f and through this b (semi-minor axis).

Helmert (1901) found from 1400 gravity values

$$\gamma_a = 9.7803 m/s^2, \beta = 0.005302$$

$$f = 1/298.3$$

Normal-potential in spherical harmonics.

$$U(\beta,\lambda,u) = U(\overline{\varphi},\lambda,r) =$$

$$\frac{GM}{r}(1 - \sum_{i=1}^{\infty} \left(\frac{a}{r}\right)^{i} J_{2i} P_{2i}(\sin\overline{\varphi}))$$

$$+ \frac{\omega^2}{2} r^2 \cos\overline{\varphi}^{\,2}$$

$$J_2 = -C_{20}$$

From this we easily calculate $\vec{\gamma} = \nabla U$,

Normalgravity

$$\gamma = \left| \nabla U \right|$$

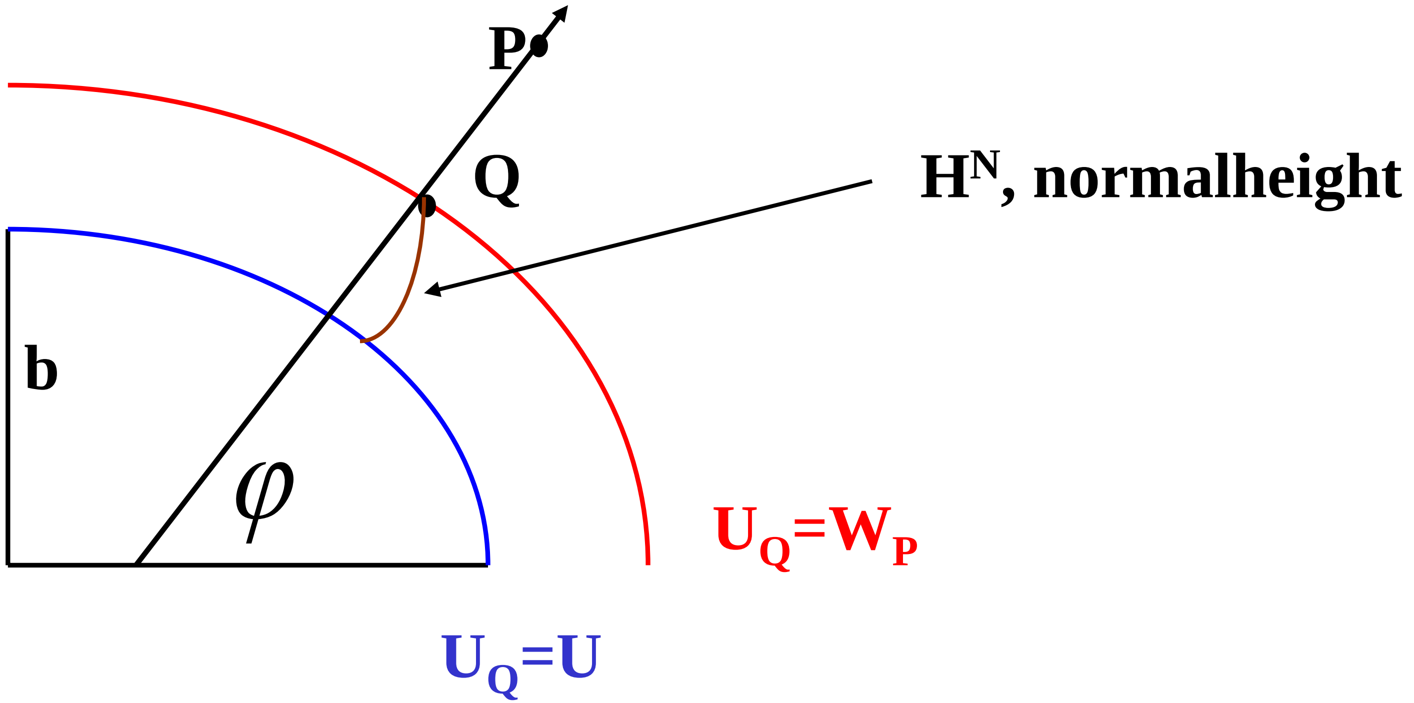
Approximative expression

$$\gamma_0(\varphi, h=0) = \gamma_a(1+\beta \sin^2\varphi + \beta_1 \sin^4\varphi \ldots)$$

$$\gamma(\varphi, h) = \gamma_0(1+\frac{2}{a}(1+f+m-2f\sin^2\varphi)h+\ldots)$$

$$m = \frac{\omega^2 a^2 b}{GM}, \beta_1 = \frac{1}{8}f^2 - \frac{5}{8}fm$$

Geometry of Normal-Gravity.



Geodetic referencesystem (GRS)

Set af parameters, which determine the ellipsoide and the normal-gravityfield.

GRS80:

a=6378137 m,

$GM=3.986004 \times 10^{14}\ m^3/s^2$

$J_2=-C_{20}=0.00108263$

$\omega=7.292115 \times 10^{-5}$ rad/s

Older systems

Hayford=International Ellipsoide:

a=6378388 m, 1/f=297,

International gravity formula 1928

Krassowsky (USSR, nu Rusland):

a=6378245.0 m, 1/f=298.3

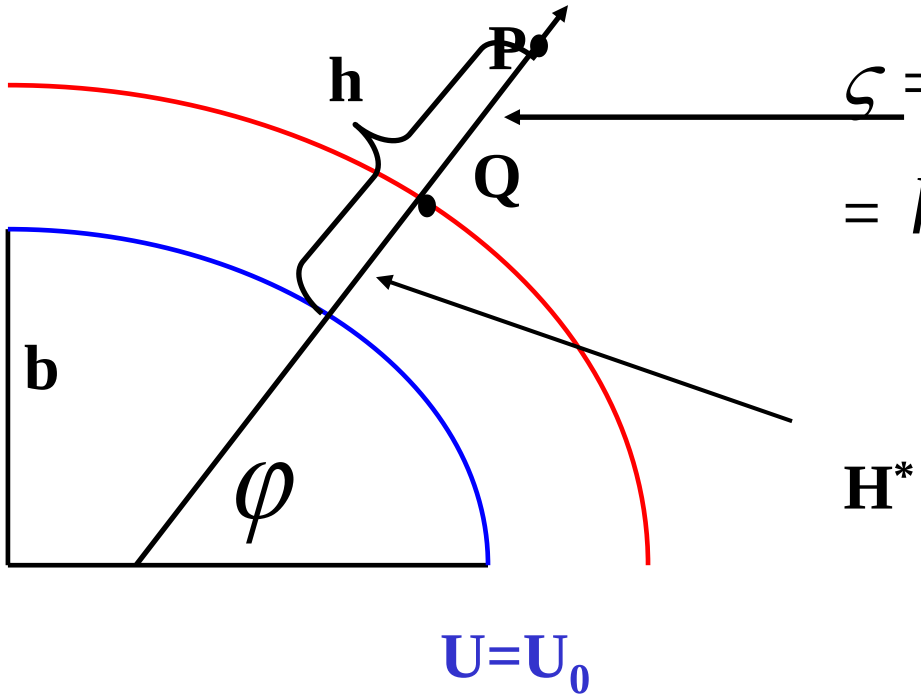
Bessel: a=6377397.0, 1/f=299.15

Clark: a=6378249 m, 1/f=293.46 (1880)

a=6378206 m, 1/f=294.979 (1866),

Centers of ellipsoids may be several 100 m wrong.

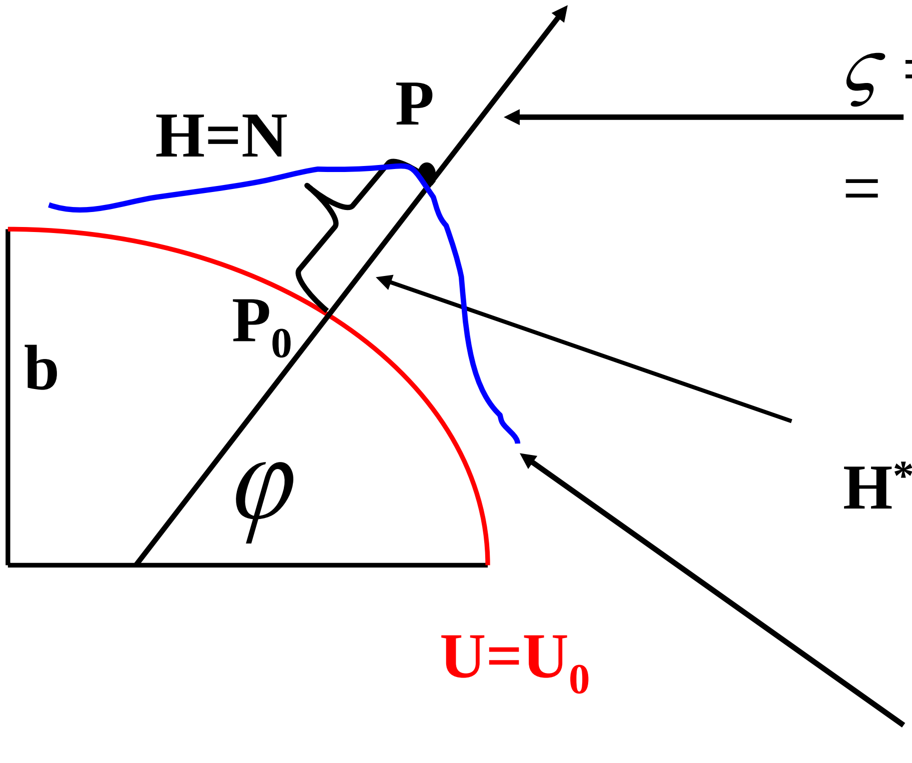
Height anomaly.



ς = *height anomaly*,

$= h - H^*$

Geoidheight, N, point P on the geoide.



$\varsigma = \textit{height anomali},$

$= N, \textit{geoidheight}$

Bruns equation (linearizing)

$$On \quad the\ geoid\ is$$

$$W(P) = W_0 = U_0 = U(P_0)$$

$$U(P) = U(P_0) + \frac{\partial U}{\partial h} N + \ldots$$

$$U(P) - W(P) = -\gamma \cdot N$$

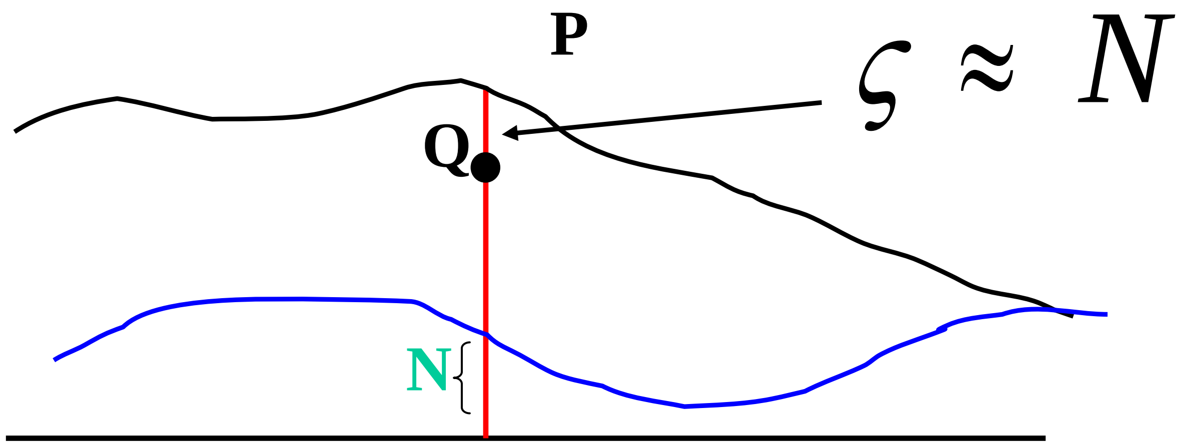
$$N = (W(P) - U(P)) / \gamma = T / \gamma$$

Height anomali

Geoid height may be generalized to an arbitrary point:

Distance between a point P and a point Q, where

W(P)=U(Q), on the same ellipsoid normal



Generalised Bruns equation.

$$\zeta(P) = (W(P) - U(Q)) / \gamma(Q)$$

$$\varsigma(P) = T / \gamma$$

If local height system different from global, then the difference (bias), N_0 must be added.

Gravity disturbance

We find the point P on the ellipsoid-normal, which have ellipsoidal height h, above the ellipsoid. Normal gravity is computed in the same point as P !

$$\delta g = g(P) - \gamma(P)$$

Gravity anomaly

We find in practice Q as the point on the ellipsoid-normal, which have ellipsidal height equal to P’s height above the geoide, H.

$$\Delta g = g(P) - \gamma(Q)$$

Gravity anomaly, linearized.

$$g(P) \approx -\frac{\partial W}{\partial r}\Big|_P$$

$$\gamma(Q) \approx -\frac{\partial U}{\partial r}\Big|_Q = -\frac{\partial U}{\partial r}\Big|_P - \frac{\partial^2 U}{\partial r^2} N$$

$$= -\frac{\partial U}{\partial r}\Big| - \frac{\partial \gamma}{\partial r}\frac{T}{\gamma} \quad where\ \gamma \approx \frac{GM}{r^2}, \frac{\partial \gamma}{\partial r} = -2\frac{GM}{r^3}$$

$$\Delta g = g(P) - \gamma(Q) =$$

$$-\frac{\partial}{\partial r}T - \frac{\partial \gamma}{\partial r}\frac{1}{\gamma}T = -\frac{\partial}{\partial r}T - \frac{2}{r}T$$