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Geodesy Course 2008. Exercise 1.

1.1. The centrifugal potential is given by

$$\Phi(r,\overline{\varphi}) = \frac{\omega^2}{2}(r\cos(\overline{\varphi}))^2 = \frac{\omega^2}{2}(X^2+Y^2)$$

Compute the value at Equator expressed in m^2/s^2. Use ω=7.292115x10^{-5} radians/s and semi-major axis a=6378137.0 m.

1.2. A point has the geodetic coordinates $\varphi = 56^o$, $\lambda = 10^o$, h = 0 m, where (X,Y,Z),

$$\overline{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3520532.589\ m \\ 620764.882\ m \\ 5264442.236\ m \end{pmatrix}$$

r= 6363477.886 m, $\overline{\varphi} = 55.82$ degrees.

What is the value of the centrifugal potential Φ ?

1.3. Calculate the gradient of Φ in the point expressed in local spherical coordinates.

1.4. The Earth is regarded as spherical and homogeneous with GM = 3.986005 * 10^{14} m^3/s^2. Calculate the **gradient** in a point with spherical coordinates as used in exercise 2.

1.5. Calculate the value of the Laplace operator applied on Φ.

1.**6.** Find the expression for $P_3(t)$ using the recursion equation

$$P_i(t) = \frac{2i-1}{i}tP_{i-1}(t) - \frac{i-1}{i}P_{i-2}(t)$$

Find the values for t=0 and t=1 using the equation.

1.7. A simple approximation to the potential W of the Earth is given by

$$W(r,\overline{\varphi},\lambda) = \frac{GM}{r}(1 - J_2\frac{a^2}{r^2}P_2(\sin\overline{\varphi})) + \frac{\omega^2}{2}\left(r\cos\overline{\varphi}\right)^2$$

with constants (GM, a, J_2, ω) from GRS80,

$$GM = 3.986005\times10^{14}\ \ [m^3/s^2]$$

$a = 6378137\ m$
$b = 6356752.314\ m$
$J_2 = 1.08263 \times 10^{-3}$
$\omega = 7.19115 \times 10^{-5}\ m$

Calculate the potential at the following points, all having the height, h, above the ellipsoid equal to zero: North pole, Equator with $\lambda = 0$ and the point in exercise 2 .

Note the order of magnitude in the variations of W between the 3 points. What is the corresponding heights, N, of the geoid in GRS80 ? Use that the GRS80 normal potential U_0 at the ellipsoid is equal to 62636860.850 m^2/s^2 and Bruns formula $N=(W-U_0)/\gamma$, with $\gamma=9.8$ m/s^2.